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A TREATISE OF PRACTICAL GEOMETRY.

IN THREE PARTS.

BY

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The late Dr. *DAVID GREGORY*,
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Tranſlated from the Latin; *with Additions.*

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ERRATA.

Page 81. Line 14. after *Glaziers* read *uſe*. Page 143. Line 17. for .0036, read .0034.

MVSEVM BRITANNICVM

ADVERTISEMENT.

THIS Treatiſe was compoſed in Latin about ſixty Years ago by Dr. David Gregory, *then Profeſſor of Mathematicks in the Univerſity of* Edinburgh, *where it has been conſtantly taught ſince that time, immediately after* Euclid's *Elements and the plain Trigonometry, as proper for exerciſing the Students in the Application of Geometry to Practice. The Bookſeller having procured an* Engliſh *Tranſlation of it, which had been made by an ingenious* Gentleman *when a Student here, this Tranſlation has been reviſed; and ſeveral Additions have been made to the Treatiſe itſelf, in order to render it more uſeful at this time. The Reader will find theſe diſtinguiſhed from the Author's Text.*

College of Edinb.
May 1. 1745.

A
TREATISE
OF
Practical Geometry.

HAving explained the firſt books of *Euclid*, with the eleventh and twelfth, which may ſerve for geometrical elements; and having alſo taught the plain Trigonometry; we are now to ſubjoin ſome corollaries which are eaſily deduced from them, that contain practical rules of great uſe in the affairs of life, concerning the menſuration of lines, angles, ſurfaces and ſolids.

This treatiſe of practical Geometry is divided into three parts. In the firſt, we treat of the menſuration of lines and angles; to which we have ſubjoined Surveying. In the ſecond, we treat of ſurfaces, not of ſuch as are plane only, but of ſome curve ſurfaces likewiſe; as of the ſurface of the cylinder, cone and ſphere; and of thoſe parts of the ſphere which we have frequently occaſion to conſider. It is ſhewn how to expreſs the area of theſe in the ſuperficial meaſures that are now in uſe amongſt us. The third part treats of ſolid figures and their menſuration. After deducing the rules for finding the ſolid content of the paralellopipedon, priſm, pyramid, cylinder, cone, &c. from *Euclid*, we add from *Archimedes* the menſuration of the ſphere and ſpheroid, and of their ſegments,

ments, demonſtrated in an eaſy manner; from whence a method is derived for finding the contents of veſſels that are either full, or in part empty, in the wet as well as the dry meaſures that are now in uſe amongſt us.

PART I.

A Line, or length to be meaſured, whether it be diſtance, height or depth, is meaſured by a line leſs than it. With us the leaſt meaſure of length is an inch; not that we meaſure no line leſs than it, but becauſe we do not uſe the name of any meaſure below that of an inch; expreſſing leſſer meaſures by the fractions of an inch; and in this treatiſe, we uſe decimal fractions as the eaſieſt. Twelve inches make a foot, three feet and an inch make the *Scotch* ell, ſix ells make a fall, forty falls make a furlong, eight furlongs make a

mile.

mile. So that the *Scotch* mile is 1184 paces, accounting every pace to be five feet. Theſe things are according to the ſtatutes of *Scotland*, notwithſtanding which the glaziers uſe a foot of only eight inches; and other artiſts for the moſt part uſe the *Engliſh* foot, on account of the ſeveral Scales marked on the *Engliſh* foot-meaſure for their uſe. But the *Engliſh* foot is ſomewhat leſs than the *Scotch*, ſo as that 185 of theſe make 186 of thoſe.

Lines, to the extremities and any intermediate point of which you have eaſy acceſs, are meaſured, by applying to them the common meaſure a number of times. But lines, to which you cannot have ſuch acceſs, are meaſured by methods taken from Geometry. The chief whereof we ſhall here endeavour to explain. The firſt is by the help of the geometrical ſquare.

" As for the *Engliſh* meaſures, the

“ yard is three feet or thirty ſix inches. A pole is ſixteen feet and a half, or five yards and a half. The chain, commonly called *Gunter's* chain, is four poles, or twenty two yards, that is ſixty ſix feet. An *Engliſh* ſtatute mile is fourſcore chains, or 1760 yards, that is 5280 feet.

“ The chain (which is now much in uſe, becauſe it is very convenient for ſurveying) is divided into a hundred links, each of which is $7\frac{92}{100}$ of an inch; whence it is eaſy to reduce any number of thoſe links to feet, or any number of feet to links.

“ A chain that may have the ſame advantages in ſurveying in *Scotland* as *Gunter's* chain has in *England*, ought to be in length ſeventy four feet, or twenty four *Scots* ells, if no regard is had to the difference of the *Scotch* and *Engliſh* foot above-mentioned. But if re-

“ gard is had to that difference, the
“ *Scotch* chain ought to conſiſt of
“ 74 $\frac{2}{5}$ *Engliſh* feet, or 74 feet, 4
“ inches, and $\frac{4}{5}$ of an inch. This
“ chain being divided into an hun-
“ dred links, each of thoſe links is
“ 8 inches, and $\frac{928}{1000}$ of an inch.”
“ In the following table the moſt
“ noted meaſures are expreſſed in
“ *Engliſh* inches and decimals of an
“ inch.

	Eng. Inch.	*Dec.*
The *Engliſh* foot is -	12	000
The *Paris* foot, -	12	788
The *Rhinland* foot, meaſured by Mr. *Picart*, -	12	362
The *Scotch* foot, -	12	065
The *Amſterdam* foot by *Snellius* and *Picart*, -	11	172
The *Dantzick* foot by *Hevelius*, - -	11	297
The *Daniſh* foot by Mr. *Picart*, - - -	12	465
The *Swediſh* foot by the ſame, - -	11	692

The

	Inch.	*Dec.*
The *Bruſſels* foot by Mr. *Picart,* - - -	10	828
The *Lyons* foot by Mr. *Auzout,* - -	13	458
The *Bononian* foot by Mr. *Caſſini,* - -	14	938
The *Milan* foot by Mr. *Auzout,* - -	15	631
The *Roman* Palm uſed by merchants, according to the ſame, -	9	791
The *Roman* palm uſed by Architects, - -	8	779
The palm of *Naples,* according to Mr. *Auzout,*	10	314
The *Engliſh* yard, -	36	000
The *Engliſh* ell, - -	45	000
The *Scotch* ell, -	37	200
The *Paris* aune uſed by Mercers, according to Mr. *Picard,* -	46	786
The *Paris* aune uſed by Drapers, according to the ſame, - -	46	680

The

	Inch.	Dec.
The *Lyons* aune by Mr. *Auzout*, - -	46	570
The *Geneva* aune, -	44	760
The *Amſterdam* ell, -	26	800
The *Daniſh* ell. by Mr. *Picart*, - -	24	930
The *Swediſh* ell, -	23	380
The *Norway* ell, -	24	510
The *Brabant* or *Antwerp* ell,	27	170
The *Bruſſels* ell, -	27	260
The *Bruges* ell, -	27	550
The brace of *Bononia* according to *Auzout*, -	25	200
The brace uſed by architects in *Rome*, -	30	730
The brace uſed in *Rome* by merchants, -	34	270
The *Florence* brace uſed by merchants, according to *Picart*. - -	22	910
The *Florence* geographical brace, - -	21	570
The vara of *Seville*, -	33	127
The vara of *Madrid*, -	39	166

The vara of *Portugal*,	44	031
The cavedo of *Portugal*,	27	354
The antient *Roman* foot,	11	632
The *Perſian* ariſh, according to Mr. *Greaves*,	38	364
The ſhorter pike of *Conſtantinople*, according to the ſame, -	25	576
Another pike of *Conſtanſtinople*, according to Meſſrs. *Mallet* and *De la Porte*, - -	27	920

PROPOSITION I.

PROBLEM I.

To deſcribe the ſtructure of the geometrical ſquare.

THE geometrical ſquare is made of any ſolid matter, as braſs or wood, or of any four plain rulers joined together at right angles, (as in fig. 1.) where A is the centre, from which hangs a thread with a ſmall weight at the end, ſo as to be directed always to the centre. Each of the ſides, BE, and DE, is divided into an hundred equal parts, or (if the ſides be long enough to admit of it) into a thouſand parts; C, and F, are two ſights fixed on the ſide AD. There is moreover an index GH, which, when there is occaſion, is joined to the centre A in ſuch manner as that it can move round, and remain in any given ſituation. On this index are two ſights perpendicular to the right line going from the centre of the inſtrument; theſe are K and L. The ſide DE of the inſtrument is called the upright ſide, BE the reclining ſide.

PROPO-

PROPOSITION II. FIG. 2.

To meaſure an acceſſible height, AB, *by the help of a geometrical ſquare, its diſtance being known.*

LET BR be an horizontal plain, on which there ſtands perpendicularly any line AB: Let BD, the given diſtance of the obſervator from the height, be 96 feet; let the height of the obſervator's eye be ſuppoſed 6 feet; and let the Inſtrument held by a ſteady hand, or rather leaning on a ſupport, be directed towards the ſummit A, ſo that one eye, (the other being ſhut,) may ſee it clearly thro' the ſights; the perpendicular or plum-line mean while hanging free, and touching the ſurface of the inſtrument: Let now the perpendicular be ſuppoſed to cut off on the right ſide KN 80 equal parts: It is clear that LKN, ACK, are ſimilar triangles; for the angles LKN, ACK, are right angles, and therefore equal; moreover LN and AC are paralell, as being both perpendicular to the horizon; conſequently, by *prop.* 29. 1. *B. of Euclid.* the angles KLN, KAC are equal; wherefore by the ſecond corrolary of the 32. *prop.* 1. *B.* of *Euclid,* the angles LNK, and AKC, are likewiſe equal: So that in the triangles NKL, KAC (by the 4. *prop.* of the 6. *B.* of *Euclid)* as NK : KL : : KC (*i. e.* DB) : CA; that is, as 80 to 100, ſo is 96 feet to CA. Therefore, by the rule of three, CA will be found to be 120 feet; and CB, which is 6 feet, being added, the whole height is 126 feet.

But if the diſtance of the obſervator from the height, as BE, be ſuch that when the inſtrument is directed as formerly toward the ſummit A, the perpendicular fall on the angle P, oppoſite to H, the centre of the inſtrument; and BE or CG be given of 120 feet, CA will alſo be 120 feet. For in the triangles HGP, ACG, æqui-angular, as in the preceeding caſe, as PG : GH : : GC : GA. But PG is equal to GH; therefore GC is likewiſe equal to CA. That is, CA will be 120 feet, and the whole height 126 feet, as before.

Let the diſtance BF be 300 feet, and the perpendicular or plum-line cut off 40 equal parts from the reclining ſide: Now in this caſe the angles QAC, QZI, are equal, by the 29. *prop.* 1. *B.* of *Euclid.* And by the ſame *prop.* the angles QZI, ZIS are equal; therefore the angle ZIS is equal to the angle QAC. But the angles ZSI, QCA are equal, being right angles. Therefore in the æquiangular triangles ACQ, SZI, by the 4. *prop.* of the 6. *B.* of *Euclid*, it will be as ZS : SI : : CQ : CA. That is, as 100 to 40, ſo is 300 to CA. Wherefore, by the rule of three, CA will be found to be of 120 feet. And by adding the height of the obſervator, the whole BA will be 126 feet. Note, that the height is greater than the diſtance, when the perpendicular cuts the right ſide, and leſs, if it cut the reclined ſide: And that the height and diſtance are equal, if the perpendicular fall on the oppoſite angle.

SCHOLIUM. FIG. 3.

If the height of a tower, to be meaſured as above, end in a point, as in fig. 3. the diſtance of the obſervator oppoſite to it, is not CD, but is to be accounted to the perpendicular from the point A; that is, to CD muſt be added the half of the thickneſs of the tower, viz. BD: Which muſt likewiſe be underſtood in the following propoſitions, when the caſe is ſimilar.

PROPOSITION III. PROB. FIG. 4.

From the height of a tower AB given, to find a diſtance on the horizontal plain B C, by the geometrical ſquare.

LET the inſtrument be ſo placed as that the mark C in the oppoſite plain may be ſeen through the ſights, and let it be obſerved how many parts are cut off by the perpendicular. Now by what hath been already demonſtrated, the triangles AEF, ABC, are ſimilar; therefore, by 4th, 6. *Eucl.* it will be as EF to AE, ſo AB (compoſed of the height of the Tower BG, and of the height of the centre of the inſtrument A, above the tower AG) to the diſtance BC. Wherefore, if by the rule of three, you ſay, as EF to AE, ſo is AB to BC, it will be the diſtance ſought.

PROPOSITION IV. FIG. 5.

To meaſure any diſtance at land or ſea, by the geometrical ſquare.

IN this operation the index is to be applied to the inſtrument, as was ſhown in the deſcription; and by the help of a ſupport, the inſtrument is to be placed horizontally at the point A; then let it be turn'd till the remote point F, whoſe diſtance is to be meaſured, be ſeen through the fixed ſights: and bringing the index to be parallel with the other ſide of the inſtrument, obſerve by the ſights upon it any acceſſible mark B, at a ſenſible diſtance: then carrying the inſtrument to the point B, let the immoveable ſights be directed to the firſt ſtation A, and the ſights of the index to the point F. If the index cut the right ſide of the ſquare, as in K, in the two triangles BRK, and BAF, which are æquiangular, it will be (by 4th 6. *Eucl.*) as BR to RK, ſo BA (the diſtance of the ſtations to be meaſured with a chain) to AF; and the diſtance AF ſought, will be found by the rule of three. But if the index cut the reclined ſide of the ſquare in any point L, where the diſtance of a more remote point is ſought; in the triangles BLS, BAG, the ſide LS ſhall be to SB, as BA to AG the diſtance ſought, which accordingly will be found by the rule of three.

PROPOSITION V.

PROB. FIG. 6.

To meaſure an acceſſible height by means of a plain mirror.

LEt AB be the height to be meaſured; let the mirror be placed at C, in the horozontal plain BD at a known diſtance BC: let the obſerver go back to D, till he ſee the image of the ſummit in the mirror, at a certain point of it which he muſt diligently mark, and let DE be the height of the obſervator's eye. The triangles ABC and EDC are æquiangular: For the angles at D and B are right angles; and ACB, ECD, are equal, being the angles of incidence and reflection of the ray AC, as is demonſtrated in optics; wherefore the remaining angles at A, and E, are alſo equal: therefore, by 4th, 6. *Eucl.* it will be, as CD to DE, ſo CB to BA; that is, as the diſtance of the obſervator from the point of the mirror in the right line betwixt the obſervator and the height, is to the height of the obſervator's eye, ſo is the diſtance of the tower from that point of the mirror, to the height of the tower ſought; which therefore will be found by the rule of three.

Note 1ſt, The obſervator will be more exact if at the point D, a ſtaff be placed in the ground perpendicularly, over the top of which the obſervator may ſee a point of the glaſs exactly in a line betwixt him and the tower.

Note 2d, In place of a mirror may be uſed the ſurface of water contained in a veſſel, which naturally becomes parallel to the horizon.

PROPOSITION VI. Fig. 7.

To meaſure an acceſſible height A B by means of two ſtaves.

LET there be placed perpendicularly in the ground a longer ſtaff DE, likewiſe a ſhorter one FG, ſo as the obſervator may ſee A, the top of the height to be meaſured, over the ends D, F, of the two ſtaves; let FH and DC parallel to the horizon meet DE and AB in H and C, then the triangles FHD, DCA, ſhall be æquiangular; for the angles at C, and H, are right ones, likewiſe the angle A, is equal to the angle FDH, by 29. 1. *Eucl.* wherefore the remaining angles DFH, and ADC, are alſo equal: wherefore by 4. 6. *Eucl.* as FH, the diſtance of the ſtaves, to HD the exceſs of the longer ſtaff above the ſhorter, ſo is DC the diſtance of the longer ſtaff from the tower, to CA the exceſs of the height of the tower above the longer ſtaff. And thence CA will be found by the rule of three.

To which if the length DE be added, you will have the whole height of the tower BA, Q. E. F.

SCHOLIUM Fig. 8.

Many other methods may be occaſionally contrived for meaſuring an acceſſible height. For example, from the given length of the ſhadow

BD, I find out the height AB thus; Let there be erected a ſtaff CE perpendicularly, producing the ſhadow EF: the triangles ABD, CEF, are æquiangular, for the angles at B, and E, are right; and the angles ADB, and CFE, are equal, each being equal to the angle of the ſun's elevation above the horizon; therefore, by 4. 6. *Eucl.* as EF the ſhadow of the ſtaff, to EC the ſtaff itſelf, ſo BD the ſhadow of the tower, to BA the height of the tower; tho' the plain on which the ſhadow of the tower falls be not parallel to the horizon, if the ſtaff be erected in the ſame plain, the rule will be the ſame.

PROPOSITION VII.

To meaſure an inacceſſible height by means of two ſtaves.

HItherto we have ſuppoſed the height to be acceſſible, or that we can come at the lower end of it: now if, becauſe of ſome impediment, we cannot get to a tower, or if the point whoſe height is to be found out, be the ſummit of a hill, ſo that the perpendicular be hid within the hill; if, I ſay, for want of better inſtruments, ſuch an inacceſſible height is to be meaſured by means of two ſtaves, let the firſt obſervation be made with the ſtaves DE and FG as in Prop. I. then the obſervator is to go off in a direct line from the the height and firſt ſtation, till he come to the ſecond ſtation, where he is to place the longer ſtaff perpendicularly at RN, and the ſhorter ſtaff at KO, ſo that the ſummit A may be ſeen along their tops, that is, ſo that the points KNA may be in the ſame right line. Through the point N let there be drawn the right line NP parallel to FA: wherefore in the triangles KNP, KAF, the angles KNP, KAF are equal by the 29th 1. *Eucl.* alſo the angle AKF is common to both; conſequently the remaining angle KPN is equal to the remaining angle KFA. And therefore by 4th 6. *Eucl.* PN : FA :: KP : KF. But the triangles PNL, FAS, are ſimilar. Therefore, by 4th 6. *Eucl.* PN : FA :: NL : SA. Therefore by the 11th 5. *Eucl.* KP : KF :: NL : SA. Thence alternately it will be as KP (the Exceſs of the greater diſtance of the ſhort ſtaff from the long one above its leſſer diſtance from it) to NL, the exceſs of the longer Staff above the ſhorter, ſo KF, the diſtance of the two ſtations of the ſhorter ſtaff, to SA the exceſs of the height ſought above the height of the ſhorter ſtaff. Wherefore SA will be found by the rule of three. To which let the height of the ſhorter ſtaff be added, and the ſum will give the whole inacceſſible Height BA, Q. E. F.

Note 1. In the ſame manner may an inacceſſible height be found by a geometrical ſquare, or by a plain ſpeculum. But we ſhall leave the rules to be found out by the ſtudent for his own exerciſe.

Note 2. That by the height of the ſtaff we underſtand its height above the ground in which it is fixed.

Note 3. Hence depends the method of uſing other inſtruments invented by geometricians; for example of the geometrical croſs; and if all things be juſtly weighed, a like rule will ſerve for it as here: but we incline to touch only upon what is moſt material.

PROPOSITION VIII. Fig. 9.

To meaſure the diſtance AB, to one of whoſe extremities we have acceſs, by the help of four ſtaves.

LET there be a ſtaff fixed at the Point A, then going back at ſome ſenſible diſtance in the ſame right Line, let another be fixed in C, (ſo as that both the points A and B be cover'd and hid by the Staff C.) Likewiſe going off in a perpendicular from the right line CB at the point A, (the method of doing which ſhall be ſhown in the following *Scholium*) let there be placed another Staff at H; and in the right line CKG (perpendicular to the ſame CB at the point C,) and at the point of it K, ſuch that the points K, H and B may be in the ſame right line, let there be fixed a fourth ſtaff. Let there be drawn, or let there be ſuppoſed to be drawn a light line HG parallel to CA. The triangles KHG, HAB will be equiangular; for the angles HAB, KGH are right angles. Alſo by 29th 1. *Eucl.* the angles ABH, KHG are equal, wherefore by the 4th 6. *Eucl.* as KG (the exceſs of CK above AH) to GH or to CA, the diſtance betwixt the firſt and ſecond ſtaff, ſo is AH the diſtance betwixt the firſt and third ſtaff to AB the diſtance ſought.

SCHOLIUM. Fig. 10.

To draw on a plain a right line AE perpendicular to CH, from a given point A, take the right lines AB AD, on each ſide equal, and in the points B and D, let there be fixed ſtakes, to which let there be tied two equal ropes BE DE (or one having a mark in the middle) and holding in your hand their extremities joined, (or the mark in the middle, if it is but one) draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully ſtretched, let there be placed a third ſtake at E, the right line AE will be perpendicular to CH in the point A, by 11th 1. *Eucl.* In a manner not unlike to this may any problems that are reſolved by the ſquare and compaſſes, be done by ropes and a cord turned round as a radius.

PROPOSITION 9. FIG. 11.

To meaſure the diſtance AB, one of whoſe extremities is acceſſible.

FRom the point A, let the right line AC of a known length be made perpendicular to AB (by the preceeding Scholium,) likewiſe draw the right line CD perpendicular to CB meeting the right line AB in D, then by the 8th 6. *Eucl.* as DA :

AC : : AC : AB; wherefore when DA and AC are given, AB will be found by the rule of three, Q. E. F.

SCHOLIUM.

All the preceeding operations depend on the equality of ſome angles of triangles, and on the ſimilarity of the triangles ariſing from that equality. And on the ſame principles depend innumerable other operations which a Geometrician will find out of himſelf, as is very obvious. However, ſome of theſe operations require ſuch exactneſs in the work, and without it are ſo liable to errors, that, *cæteris paribus*, the following operations which are performed by a trigonometrical calculation, are to be preferred. Yet could we not omit thoſe above, being moſt eaſy in practice, and moſt clear and evident to thoſe who have only the firſt elements of Geometry. But if you

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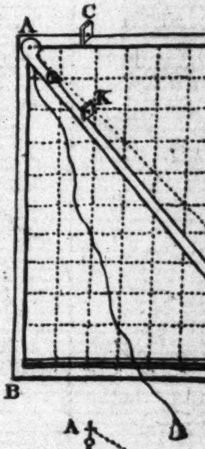

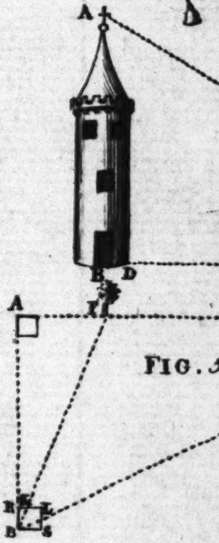

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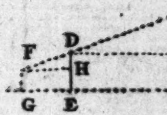

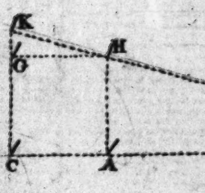

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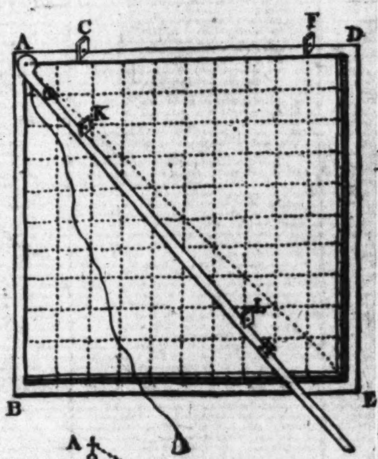

FIG. 1.

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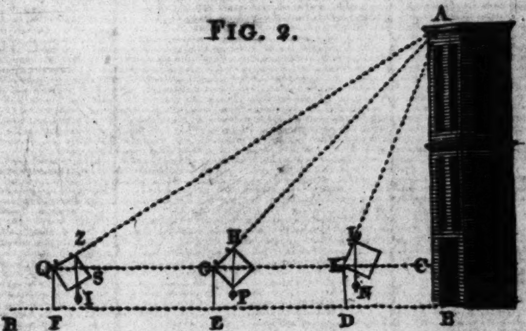

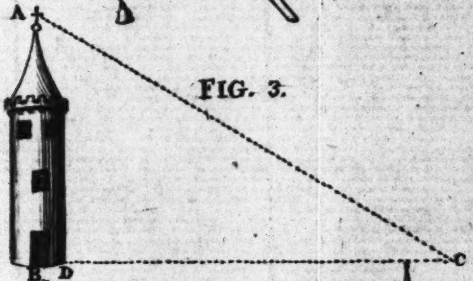

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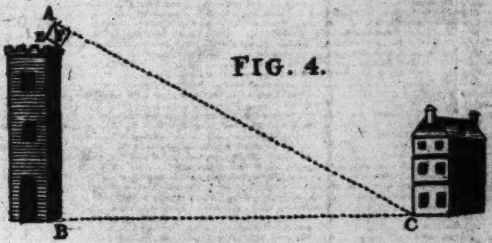

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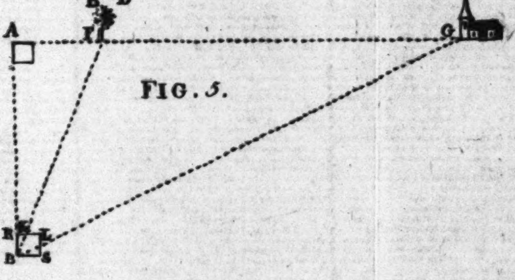
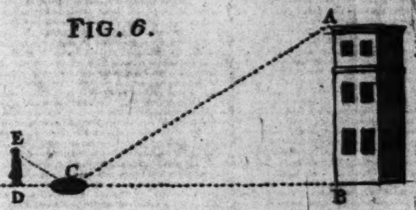


FIG. 5.

FIG. 6.



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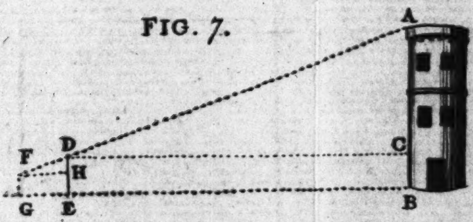

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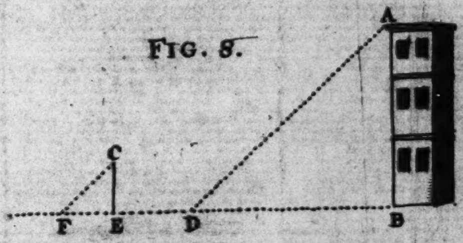


FIG 10

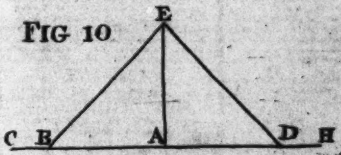


FIG. 9.

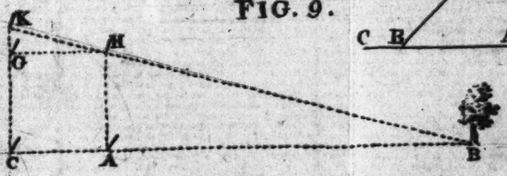
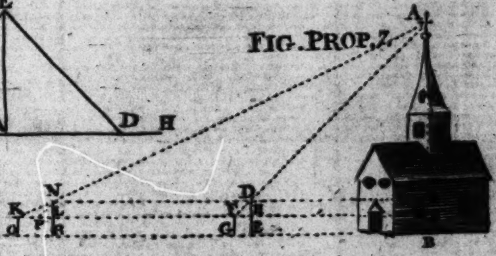


FIG. PROP. 7.



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you are provided with inſtruments, the following operations are more to be relied upon. We do not inſiſt on the eaſieſt caſes to thoſe who are skilled in plain Trigonometry, which is indeed neceſſary to any one who would apply himſelf to practice. It will be eaſy to the reader to find examples, and we have ſhown in plain Trigonometry how to find the angle or ſide of any plain triangle that is required, from the angles or ſides that may be given.

PROPOSITION X. FIG. 12.

To deſcribe the conſtruction and uſe of the geometrical quadrant.

THE geometrical quadrant is the fourth part of a circle, divided into ninety degrees, to which two ſights are adapted, with a perpendicular or plum-line hanging from the center. The general uſe of it is for in-

inveſtigating angles in a vertical plain, comprehended under right lines going from the center of the inſtrument, one of which is horizontal, and the other is directed to ſome viſible point. This inſtrument is made of any ſolid matter, as wood, copper, *&c.*

PROPOSITION XI. FIG. 13.

To deſcribe the make and uſe of the graphometer.

THE graphometer is a ſemicircle made of any hard matter, of wood, for example, or braſs, divided into 180 degrees; ſo fixed on a *fulcrum*, by means of a braſs ball and ſocket, that it eaſily turns about and retains any ſituation; two ſights are fixed on its diameter. At the center there is commonly a magnetical needle in a box. There is likewiſe a moveable ruler, which turns round the center, and retains any ſituation given it. The uſe of it is to obſerve any angle, whoſe vertex is at the center of the inſtrument in any plain, (though it is moſt commonly horizontal, or nearly ſo) and to find how many degrees it contains.

PROPOSITION XII.

FIG. 14. and 15.

To deſcribe the manner in which angles are meaſured by a quadrant or graphometer.

LET there be an angle in a vertical plain, comprehended between a line parallel to the horizon HK, and the right line RA, coming from any remarkable point of a tower or hill, or from the ſun, moon or a ſtar. Suppoſe that this angle RAH is to be meaſured by the quadrant: let the inſtrument be placed in the vertical plain, ſo as that the center A may be in the angular point, and let the ſights be directed towards the object at R, (by the help of the ray coming from it, if it be the ſun or moon, or by the help of the viſual ray, if it is any thing elſe,) the degrees and minutes in the arch BC, cut off by the perpendicular, will meaſure the angle RAH required. For from the make of the quadrant, BAD is a right angle; therefore BAR is likewiſe right, being equal to it. But becauſe HK is horizontal, and AC perpendicular, HAC will be a right angle; and therefore equal alſo to BAR. From thoſe angles ſubſtract the part HAB that is common to both; and there will remain the angle BAC equal to the angle RAH. But the arch BC is the meaſure of the angle BAC; conſequently it is likewiſe the meaſure of the angle RAH.

Note, That the remaining arch on the quadrant DC is the meaſure of the angle RAZ, comprehended between the foreſaid right line RA and AZ, which points to the Zenith.

Let it now be required to meaſure the angle ACB (Fig. 15.) in any plain, comprehended between the right lines AC and BC drawn from two points A and B, to the place of ſtation C. Let the graphometer be placed at C, ſupported by its *fulcrum* (as was ſhown above) and let the immoveable ſights on the ſide of the inſtrument DE be directed towards the point A, and likewiſe (while the inſtrument remains immoveable) let the ſights of the ruler FG (which is moveable about the center C) be directed to the point B. It is evident that the moveable ruler cuts off an arch DH, which is the meaſure of the angle ACB ſought. Moreover, by the ſame method, the inclination of DE, or of FG may be obſerved with the meridian line, which is pointed out by the magnetick needle incloſed in the box, and is moveable about the center of the inſtrument, and the meaſure of this inclination or angle found in degrees.

PROPOSITION XIII. Fig. 16.

To meaſure an acceſſible height by the geometrical quadrant.

BY the 12th Prop. of this part, let the angle C be found by means of the quadrant. Then in the triangle ABC, right angled at B (BC being ſuppoſed the horizontal diſtance of the obſervator from the tower) having the angle at C, and the ſide BC, the required height BA will be found by the third caſe of plain Trigonometry.

PROPOSITION XIV. Fig. 17.

To meaſure an inacceſſible height by the geometrical quadrant.

LET the angle ACB be obſerved with the quadrant (by the 12th prop. of this part) then let the obſerver go from C to the ſecond ſtation D, in the right line BCD (providing BCD be a horizontal plain) and after meaſuring this diſtance CD, take the angle ADC likewiſe with the quadrant. Then in the triangle ACD, there is given the angle ADC, with the angle ACD, becauſe ACB was given before: therefore (by 32d 1. *Eucl.*) the remaining angle CAD is given likewiſe. But the Side CD is likewiſe given, being the diſtance of the ſtations C and D; therefore (by the firſt caſe of oblique angled triangles in Trigonometry) the ſide AC will be found. Wherefore in the right angled triangle ABC, all the angles and the hypotenuſe AC are given; conſequently, by the 4th caſe of Trigonometry, the height ſought AB will be found; as alſo (if you pleaſe) the diſtance of the ſtation C from AB the perpendicular within the hill or inacceſſible height.

PROPOSITION XV. Fig. 18.

From the top of a given height, to meaſure the diſtance B C.

LET the angle BAC be obſerved by the 12th of this part; wherefore in the triangle ABC right-angled at B, there is given by obſervation the angle at A; whence (by the 32d 1. *Eucl.*) there will alſo be given the angle BCA; moreover the ſide AB (being the height of the tower) is ſuppoſed to be given. Wherefore by the 3d caſe of Trigonometry BC the diſtance ſought will be found.

PROPOSITION XVI. FIG. 19.

To meaſure the diſtance of two places A and B, of which one is acceſſible, by the graphometer.

LET there be erected at two points A and C, ſufficiently diſtant, two viſible ſigns; then (by the 12th of this) let the two angles BAC, BCA be taken by the graphometer. Let the diſtance of the ſtations A and C be meaſured with a chain. Then the third angle B being known, and the ſide AC being likewiſe known; therefore by the firſt caſe of Trigonometry the diſtance required AB will be found.

PROPOSITION XVII. Fig. 20.

To meaſure, by the graphometer, the diſtance of two places, neither of which is acceſſible.

LET two ſtations C and D be choſen, from each of which the places may be ſeen, whoſe diſtance is ſought; let the angles ACD, ACB, BCD, and likewiſe the angles BDC, BDA, CDA, be meaſured by the graphometer, the diſtance of the ſtations C and D be meaſured by a chain, or (if it be neceſſary) by the preceeding practice. Now in the triangle ACD, there are given two angles ACD and ADC; therefore the third CAD is likewiſe given. Moreover the ſide CD is given; therefore, by the firſt caſe of Trigonometry the ſide AD will be found; after the ſame manner in the triangle BCD from all the angles, and one ſide CD given, the ſide BD is found. Wherefore in the triangle ADB, from the given ſides DA and DB, and the angle ADB contained by them, the ſide AB (the diſtance ſought) is found by the 4th caſe of Trigonometry of oblique angled triangles.

Let it be noted, that it is not neceſſary that the points A, B, C, and D be in one plain, and that any triangle is in one plain, by 2d prop. 11th of *Eucl.*

PROPOSITION XVIII. FIG. 21.

It is required by the graphometer and quadrant to meaſure an inacceſſible height AB, placed ſo on a ſteep, that one can neither go near it in an horizontal plain, nor recede from it, as we ſuppoſed in the ſolution of the 14th prop.

LET there be choſen any ſituation as C, and another D, where let ſome mark be erected; let the angles ACD and ADC be found by the graphometer, then the third angle DAC will be known. Let the ſide CD, the diſtance of the ſtations, be meaſured with a chain, and thence (by Trigon.) the ſide AC will be found. Again in the triangle ACB, right angled at B, having found by the quadrant the angle ACB, the other angle CAB is known likewiſe; but the ſide AC in the triangle ADC is already known; therefore the height required AB will be found by the 4th caſe of right angled triangles. If the height of the tower is wanted, the angle BCF will be found by the quadrant, which being taken from the angle ACB already known, the angle ACF will remain; but the angle FAC was known before; therefore the remaining angle AFC will be known; but the ſide AC was alſo known before; therefore, in the triangle AFC, all the angles, and

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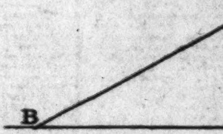
FIG. 11.



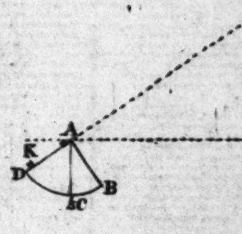
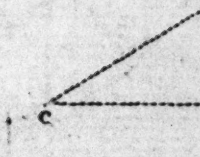
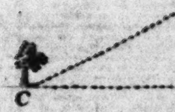
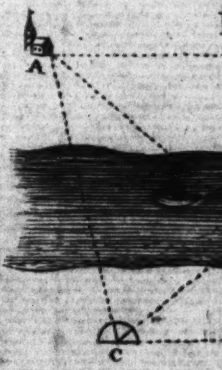


FIG. 16







Pl. II. Pag. 40.

FIG. 11.

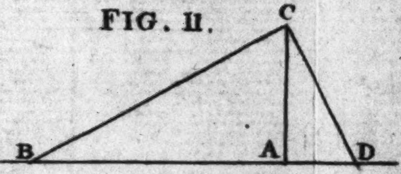


FIG. 12.

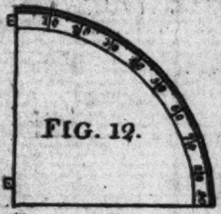


FIG. 13.

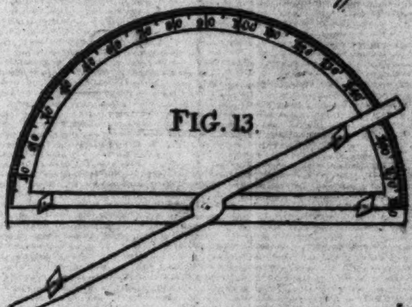


FIG. 14.

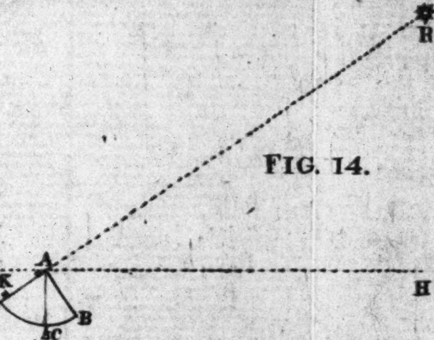
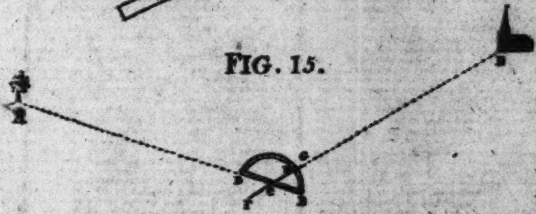


FIG. 15.



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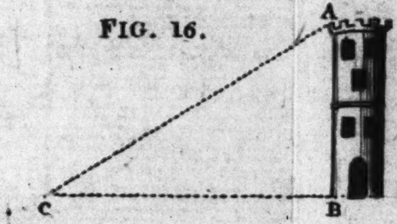


FIG. 17.

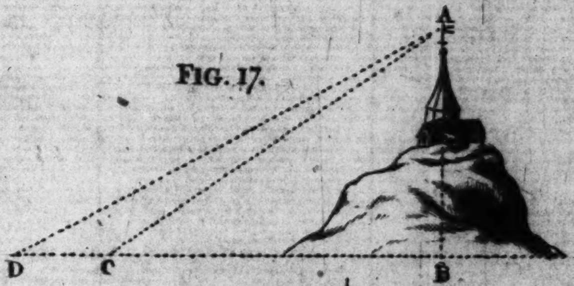


FIG. 18.

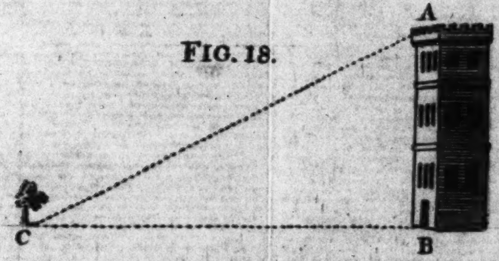


FIG. 19.

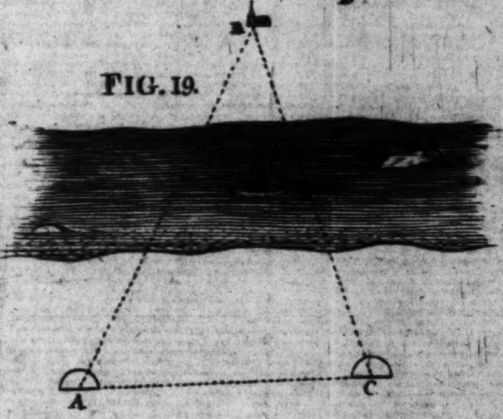


FIG 20

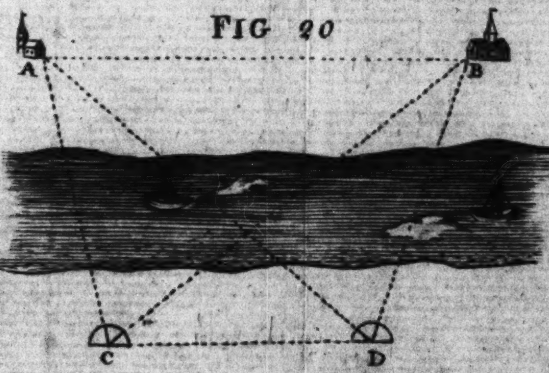
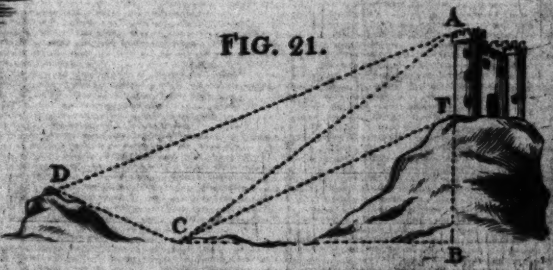


FIG. 21.



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one of the ſides AC being known, AE the height of the tower above the hill will be found by Trigonometry.

SCHOLIUM.

It were eaſy to add many other methods of meaſuring heights, and diſtances; but if what is above be underſtood, it will be eaſy (eſpecially for one that is verſ'd in the elements) to contrive methods for this purpoſe, according to the occaſion: ſo that there is no need of adding any more of this ſort. We ſhall ſubjoin here a method by which the diameter of the earth may be found out.

PROPOSITION XIX. Fig. 22.

To find the diameter of the earth from one obſervation.

LET there be choſen a high hill AB, near the ſea-ſhore, and let the obſervator on the top of it, with an exact quadrant divided into minutes and ſeconds by tranſverſe diviſions, and fitted with a teleſcope in place of the common ſights, meaſure the angle ABE contained under the right line AB, which goes to the center, and the right line BE drawn to the ſea, a tangent to the globe at E, let there be drawn from A perpendicular to BD, the line AF meeting BE in F. Now in the right angled triangle BAF all the angles are given, alſo the ſide AB, the height of the hill, which is to be found by ſome of the foregoing methods, as exactly as poſſible; and, by Trigonometry, the ſides BF and AF are found. But by corol. 36. 3d *Eucl.* AF is equal to FE; therefore BE will be known. Moreover, by 36th 3d *Eucl.* the rectangle under BA and BD is equal to the ſquare of BE. And thence by 17th 6. *Eucl.* as AB : BE : : BE : BD. Therefore, ſince AB and BE are already given, BD will be found by 11th 6. *Eucl.* or by the rule of three, and ſubſtracting BA, there will remain AD the diameter of the earth ſought.

SCHOLIUM.

Many other methods might be propoſed for meaſuring the diameter of the earth. The moſt exact in my opinion is that propoſed by Mr. *Picart*, of the academy of ſciences at *Paris*; but ſince it does not belong to this place, we refer you to the philoſophical tranſactions, where you will find it deſcribed.

“ According to Mr. *Picart*, a de-

" gree of the meridian at the lati-
" tude of 49° 21′ was 57060 *French*
" *Toiſes*, each of which contains ſix
" feet of the ſame meaſure; from
" which it follows, that if the earth
" be an exact ſphere, the circumfe-
" rence of a great circle of it will
" be 123,249,600 *Paris* feet, and
" the ſemidiameter of the earth
" 19,615,800 feet: but the *French*
" Mathematicians who have exami-
" ned Mr. *Picart*'s operations of
" late aſſure us, That the degree
" in that latitude is 57,183 *Toiſes.*
" They meaſured a degree in *Lap-*
" *land*, in the latitude of 66° 20′
" and found it of 57,438 *Toiſes.*
" By comparing theſe degrees, as
" well as by the obſervations on
" pendulums, and the theory of
" gravity; it appears that the
" earth is an oblate ſpheroid; and
" (ſuppoſing thoſe degrees to be
" accurately meaſured) the axis or
" diameter that paſſes through the

“ poles will be to the diameter of the equator as 177 to 178, or, the the earth will be 22 miles higher at the equator than at the poles. A degree has likewiſe been meaſured at the equator, and found to be conſiderably leſs than at the latitude of *Paris*; which confirms the oblate figure of the earth; but an account of this laſt menſuration has not been publiſhed as yet. If the earth was of an uniform denſity from the ſurface to the center, then, according to the theory of gravity, the meridian would be an exact ellypſis, and the axis would be to the diameter of the equator as 230 to 231; and the difference of the ſemidiameter of the equator and ſemi-axis about 17 miles.

In what follows, a figure is often to be laid down on paper, like to another figure given; and becauſe this likeneſs conſiſts in the equality of their angles, and in the ſides having the ſame proportion to each other (by the definitions of the 6th of *Eucl.*) we are now to ſhow what methods practical Geometricians uſe for making on paper an angle equal to a given angle, and how they conſtitute the ſides in the ſame proportion. For this purpoſe they make uſe of a protractor, (or, when it is wanting, a line of chords) and of a line of equal parts.

PROPOSITION XX.

FIG. 23, 24, 25, 26, and 27.

To deſcribe the conſtraction and uſe of the protractor, of the line of chords, and of the line of equal parts.

THE protractor is a ſmall ſemicircle of braſs, or ſuch ſolid matter. The ſemi-circumference is divided into 180 degrees. The uſe of it is to draw angles on any plain, as on paper, or to examine the extent of angles already laid down. For this laſt purpoſe, let the ſmall point in the center of the protractor be placed above the angular point, and let the ſide AB coincide with one of the ſides that contain the angle propoſed; the number of degrees cut off by the other ſide, computing on the protractor from B, will ſhow the quantity of the angle that is to be meaſured.

But if an angle is to be made of a given quantity on a given line, and at a given point of that line, let AB coincide with the given line, and let the center A of the inſtrument be applied to that point. Then let there be a mark made at the given number of degrees; and a right line drawn from that mark to the given point will conſtitute an angle, with the given right line of the quantity required; as is manifeſt.

This is the moſt natural and eaſy method, either for examining the extent of an angle on paper, or for deſcribing on paper an angle of a given quantity.

But when there is ſcarcity of inſtruments, or becauſe a line of chords is more eaſily carried about, (being deſcribed on a ruler on which there are many other lines beſides) practical Geometricians frequently make uſe of it. It is made thus: let the quadrant of a circle be divided into 90 degrees, as in fig. 24. the right line AB is the chord of 90 degrees; the chord of every arch of the quadrant is transferred to this line AB, which is always marked with the number of degrees in the correſponding arch.

Note, That the chord of 60 degrees is equal to the radius, by corol. 15. 4th *Eucl.* If now a given angle

EDF is to be meaſured by the line of chords, from the center D, with the diſtance DG (the chord of 60 degrees,) deſcribe the arch GH, and let the points G and H be marked where this arch interſects the ſides of the angle. Then if their diſtance GH, applied on the line of chords from A to B, gives (for example) 25 degrees, this ſhall be the meaſure of the angle propoſed.

When an obtuſe angle is to be meaſured with this line, let its complement to a ſemicircle be meaſured, and thence it will be known. It were eaſy to transfer to the diameter of a circle the chords of all arches to the extent of a ſemicircle, but ſuch are rarely found marked upon rules.

But now if an angle of a given quantity, ſuppoſe of 50 degrees, is to be made at a given point M of the right line KL (fig. 26.) From the center M, and the diſtance MN equal to the chord of 60 degrees, deſcribe the arch QN. Take off an arch NR, whoſe chord is equal to that of 50 degrees on the line of chords; join the points M and R, and it is plain that MR ſhall contain an angle of 50 degrees with the line KL propoſed.

But ſometimes we cannot produce the ſides, till they be of the length of a chord of 60 degrees on our ſcale; in which caſe it is fit to work by a circle of proportions (that is a a ſector) by which an arch may be made of a given number of degrees to any radius.

The quantities of angles are likewiſe determined by other lines uſually marked upon rules, as the lines of ſines, tangents and ſecants; but as theſe methods are not ſo eaſy or ſo proper in this place, we omit them.

To delineate figures ſimilar or like to others given, beſides the equality of the angles, the ſame proportion is to be preſerved among the ſides of the figure that is to be delineated, as is among the ſides of the figure given. For which purpoſe, on the rules uſed by artiſts, there is a line divided into equal parts, more or leſs in number, and greater or leſſer in quantity, according to the pleaſure of the maker.

A Foot is divided into inches, and an inch, by means of tranſverſe lines, into 100 equal parts; ſo that with this ſcale, any number of inches below twelve, with any part of an inch, can be taken by the compaſſes, providing ſuch part be greater than one 100th part of an inch. And this exactneſs is very neceſſary in delineating the plans of houſes, and in other caſes.

PROPOSITION XXI. FIG. 28.

To lay down on paper, by the protractor, or line of chords, and line of equal parts, a right lined figure like to one given, providing the angles and ſides of the figure given be known by obſervation or menſuration.

FOR example, ſuppoſe that it is known that in a quadrangular figure, one ſide is of 235 feet, that the angle contained by it and the ſecond ſide is of 84°, the ſecond ſide of 288 feet, the angle contained by it and the third ſide of 72°, and that the third ſide is 294 feet. Theſe things being given, a figure is to be drawn on paper like to this quadrangular figure. On your paper at a proper point A let a right line be drawn, upon which take 235 equal parts, as AB. The part repreſenting a foot is taken greater or leſſer, according as you would have your figure greater or leſs. In the adjoining figure, the 100th part of an inch is taken for a foot. And accordingly an inch divided into 100 parts, and annexed to the figure is called a ſcale of 100 feet. Let there be made at the point B, (by the preceeding prop.) an angle ABC of 84°, and let BC be taken of 288 parts like to the former. Then let the angle BCD be made of 72°, and the ſide CD of 294 equal parts. Then let the ſide AD be drawn; and it will compleat the figure like to the figure given. The meaſures of the angles A and D can be known by the protractor or line of chords, and the ſide AD by the line of equal parts; which will exactly anſwer to the correſponding angles and to the ſide of the primary figure.

After the very ſame manner, from the ſides and angles given, which bound any right lined figure, a figure like to it may be drawn, and the reſt of its ſides and angles be known.

COROLLARY.

Hence any trigonometrical problem in right lined triangles, may be reſolved by delineating the triangle from what is given concerning it, as in this propoſition. The unknown ſides are examined by a line of equal parts, and the angles by a protractor or line of chords.

PROPOSITION XXII. PROB.

The diameter of a circle being given, to find its circumference nearly.

THE periphery of any polygon inſcribed in the circle is leſs than the circumference, and the periphery of any polygon deſcribed about a circle is greater than the circumference. Whence *Archimedes* firſt diſcovered that the diameter was in proportion to the circumference, as 7 to 22 nearly; which ſerves for common uſe. But the moderns have computed the proportion of the diameter to the circumference to greater exactneſs. Suppoſing the diameter 100, the periphery will be more than 314, but leſs than 315*. But *Luctolphus van Ceulen* exceeded the labours of all; for by immenſe ſtudy he found, that ſuppoſing the diameter

100,000,000,000,000,000,000,000,000,000,000

the periphery will be leſs than

314,159,265,358,979,323,846,264,338,327,951,

but greater than

314,159,265,358,979,323,846,264,338,327,950

whence it will be eaſy, any part of the circumference being given in degrees and minutes, to aſſign it in parts of the diameter.

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* *The diameter is more nearly to the circumference, as* 113 *to* 355.

Of ſurveying and meaſuring of land.

Hitherto we have treated of the meaſuring of angles and ſides, whence it is abundantly eaſy to lay down a field, a plain, or an entire country: for to this nothing is requiſite but the protraction of triangles, and of other plain figures, after having meaſured their ſides and angles: but as this is eſteemed an important part of practical Geometry, we ſhall ſubjoin here an account of it, with all poſſible brevity, ſuggeſting withall, that a ſurveyor will improve himſelf more by one day's practice, than by a great deal of readding.

PROPOSITION XXIII. PROB.

To explain what ſurveying is, and what inſtruments ſurveyors uſe.

FIrſt, it is neceſſary that the ſurveyor view the field that is to be meaſured, and inveſtigate its ſides and angles, by means of an iron chain (having a particular mark at each foot of length, or at any number of feet as may be moſt convenient for reducing lines or ſurfaces to the received meaſures*) and the graphometer deſcribed above. *Secondly*, It is neceſſary to delineate the field *in plano*, or to form a map of it; that is, to lay down on paper a figure ſimilar to the field, which is done by the protractor, (or line of chords) and of the line of equal parts

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* *See above p. 5. the account of Gunter's chain, and of the chain that is moſt convenient for meaſuring land in Scotland.*

parts. *Thirdly*, It is neceſſary to find out the area of the field ſo ſurveyed and repreſented by a map. Of this laſt we are to treat below in the ſecond part.

The ſides and angles of ſmall fields are ſurveyed by the help of a plain table, which is generally of an oblong rectangular figure, and ſupported by a *fulcrum*, ſo as to turn every way by means of a ball and ſocket. It has a moveable frame, which ſurrounds the board, and ſerves to keep a clean paper put on the board cloſe and tight to it. The ſides of the frame facing the paper are divided into equal parts every way. The board hath beſides, a box with a magnetick needle, and moreover a large index with two ſights. On the edge of the frame of the board are marked degrees and minutes, ſo as to ſupply the room of a graphometer.

PROPOSITION XXIV.

PROB. FIG. 29.

To delineate a field by the help of a plain table, from one ſtation whence all its angles may be ſeen, and their diſtances meaſured by a chain.

LET the field that is to be laid down be ABCDE. At any convenient place F, let the plain table be erected, cover it with clean paper, in which let ſome point near the middle repreſent the ſtation. Then applying at this place the index with the ſights, direct it ſo as that through the ſights ſome mark may be ſeen at one of the angles, ſuppoſe A, and from the point F, repreſenting the ſtation, draw a faint right line along the ſide of the index, then, by the help of the chain, let FA the diſtance of the ſtation from the foreſaid angle be meaſured. Then taking what part you think convenient for a foot or pace from the line of equal parts, ſet off on the faint line the parts correſponding to the line FA that was meaſured, and let there be a mark made repreſenting the angle of the field, A. Keeping the table immoveable, the ſame is to be done with the reſt of the angles; then right lines joining thoſe marks ſhall include a figure like to the field, as is evident from 5th 6, *Eucl.*

COROLLARY.

The ſame thing is done in like manner by the graphometer; for having obſerved in each of the triangles, AFB, BFC, CFD, *&c.* the angle at the ſtation F, and having meaſured the lines from the ſtation to the angles of the field, let ſimilar triangles be protracted on paper (by the 21. of this) having their common vertex in the point of ſtation. All the lines, excepting thoſe which repreſent the ſides of the field, are to be drawn faint or obſcure.

Note 1. When a ſurveyor wants to lay down a field, let him place diſtinctly in a regiſter all the obſervations of the angles, and the meaſures of the ſides, until at time and place convenient, he draw out the figure on paper.

Note 2. The obſervations made by the help of the graphometer are to be examined; for all the angles about the point F ought to be equal to four right ones by 13th 1. *Eucl.*

PRO-

PROPOSITION XXV.

PROB. FIG. 30.

To lay down a field by means of two ſtations, from each of which all the angles can be ſeen, by meaſuring only the diſtance of the ſtations.

LET the inſtrument be placed at the ſtation F; and having choſen a point repreſenting it upon the paper which is laid upon the plain table, let the index be applied at this point, ſo as to be moveable about it. Then let it be directed ſucceſſively to the ſeveral angles of the field; and when any angle is ſeen through the ſights, draw an obſcure line along the ſide of the index. Let the index, with the ſights, be directed after the ſame manner to the ſtation G; on the obſcure line drawn along its ſide, pointing to A, ſet off from the ſcale of equal parts a line correſponding to the meaſured diſtance of the ſtations, and this will determine the point G. Then remove the inſtrument to the ſtation G, and applying the index to the line repreſenting the diſtance of the ſtations, place the inſtrument ſo that the firſt ſtation may be ſeen through the ſights. Then the inſtrument remaining immoveable, let the index be applied at the point repreſenting the ſecond ſtation G, and be ſucceſſively directed by means of its ſights, to all the angles of the field, drawing (as before) obſcure lines; and the interſection of the two obſcure lines that were drawn to the ſame angle from the two ſtations will always repreſent that angle on the plan. Care muſt be taken that thoſe lines be not miſtaken for one another. Lines joining thoſe interſections will form a figure on the paper like to the field.

SCHOLIUM.

It will not be difficult to do the ſame by the graphometer, if you keep a diſtinct account of your obſervations of the angles made by the line joining the ſtations and the lines drawn from the ſtations to the reſpective angles of the field. And this is the moſt common manner of laying down whole countries. The tops of two mountains are taken for two ſtations, and their diſtance is either meaſured by ſome of the methods mentioned above, or is taken according to common repute. The ſights are ſucceſſively directed towards cities, churches, villages, forts, lakes, turnings of rivers, woods, *&c.*

Note. The diſtance of the ſtations ought to be great enough, with reſpect to the field that is to be meaſured, ſuch ought to be choſen as are not in a line with any angle of the field. And care ought to be taken likewiſe that the angles, for example FAG, FDG, &c, be neither very acute, nor very obtuſe. Such angles are to be avoided as much as poſſible; and this admonition is found very uſeful in practice.

PROPOSITION XXVI.

PROB. FIG. 31.

To lay down any field, however irregular its figure may be, by the help of the graphometer.

LET ABCEDHG be ſuch a field. Let its angles (in going round it) be obſerved with a graphometer (by the 12th of this) and noted down; let its ſides be meaſured with a chain, and, (by what was ſaid on the 21ſt of this) let a figure like to the given field be protracted on paper. If any mountain is in the circumference, the horizontal line hid under it is to be taken for a ſide, which may be found by two or three obſervations, according to ſome of the methods deſcribed above; and its place on the map is to be diſtinguiſhed by a ſhade, that it may be known a mountain is there.

If not only the circumference of the field is to be laid down in the plan, but alſo its contents, as villages, gardens, churches, publick roads, we muſt proceed in this manner.

Let there be (for example) a church F, to be laid down on the plan. Let the angles ABF, BAF be obſerved and protracted on paper in their proper places, the interſection of the two ſides BF and AF will give the place of the church on the paper; or more exactly, the lines BF, AF being meaſured, let circles be deſcribed from the centers B and A, with parts from the ſcale correſponding

ing to the diſtances BF and AF, and the place of the church will be at their interſection.

Note 1. While the angles obſerved by the graphometer are taken down, you muſt be careful to diſtinguiſh the external angles as E and G, that they may be rightly protracted afterwards on paper.

Note 2. Our obſervations of the angles may be examined by computing if all the internal angles make twice as many right angles, four excepted, as there are ſides of the figure: for this is demonſtrated by 32d 1. *Eucl.* But in place of any external angle DEC, its compliment to a circle is to be taken.

PROPOSITION XXVII.

PROB. FIG. 32.

To lay down a plain field without inſtruments.

IF a ſmall field is to be meaſured, and a map of it to be made, and you are not provided with inſtruments; let it be ſuppoſed to be divided into triangles, by right lines, as in the figure, and after meaſuring the three ſides of any of the triangles, for example of ABC, let its ſides be laid down from a convenient ſcale on paper, by the 22d of this. Again, let the other two ſides BD, CD of the triangle CBD be meaſured and protracted on the paper by the ſame ſcale as before. In the ſame manner proceed with the reſt of the triangles of which the field is compoſed, and the map of the field will be per-

XVII.

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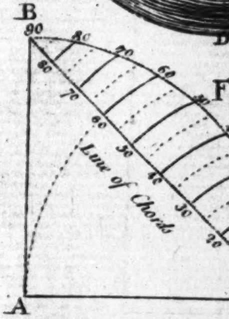
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80
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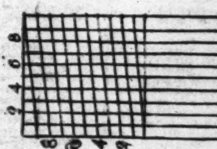
60
50
40
30
20
F
Line of Chords
A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72.°
C
22

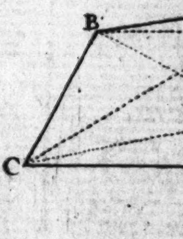
B
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FIG. 22.

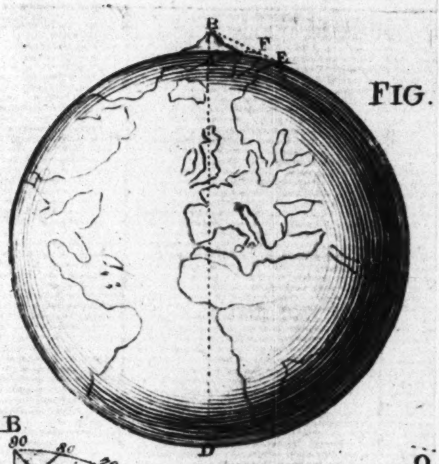


FIG. 23.

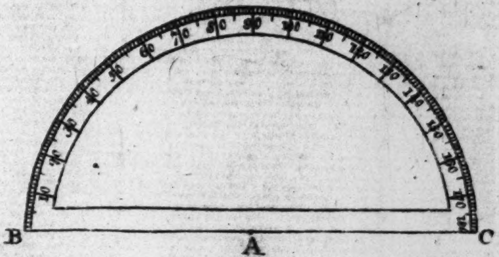


FIG. 24.

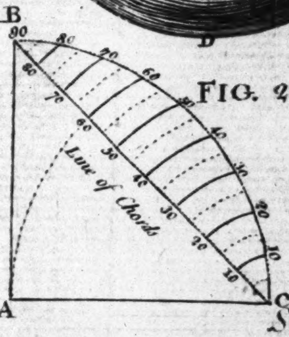


FIG. 26.

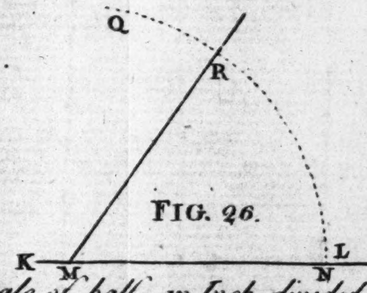
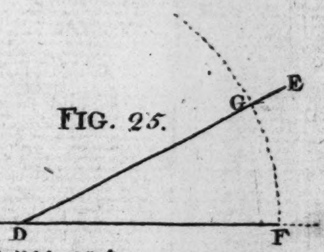
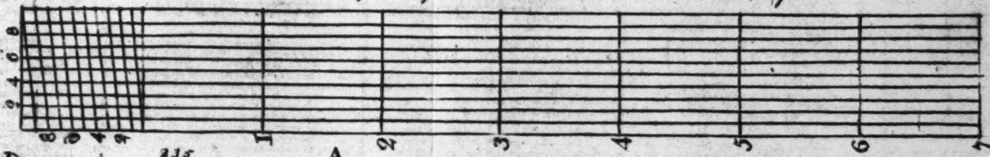


FIG. 25.



Scale of half an Inch divided into 100 parts



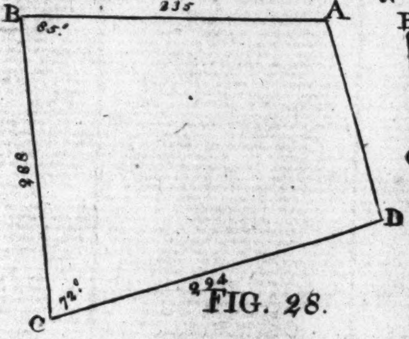


FIG. 28.

FIG. 29.

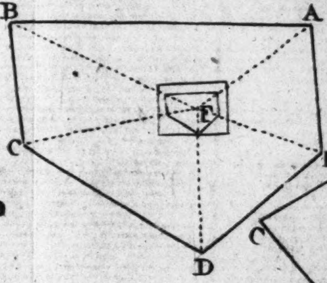


FIG. 31.

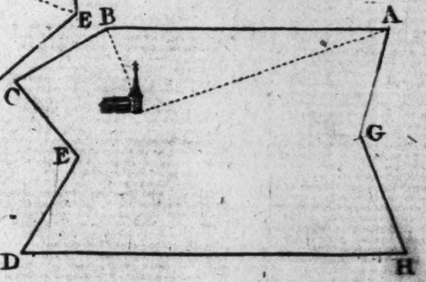


FIG. 30.

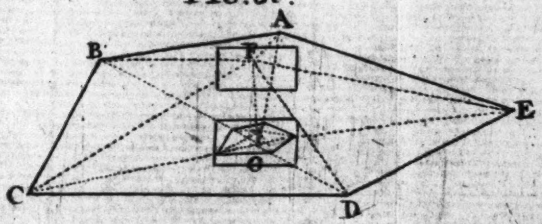
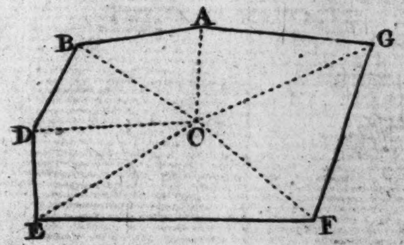


FIG. 32.



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perfected; for the three ſides of a triangle determine the triangle; whence each triangle on the paper is ſimilar to its correſpondent triangle in the field, and is ſimilarly ſituated; conſequently the whole figure is like to the whole field.

SCHOLIUM.

If the field be ſmall, and all its angles may be ſeen from one ſtation, it may be very well laid down by the plain table by the 24th of this. If the field be larger, and have the requiſite conditions, and great exactneſs is not expected, it likewiſe may be plotted by means of the plain table, or by the graphometer, according to the 25th of this; but in fields that are irregular and mountainous, when an exact map is required we are to make uſe of the graphometer as in the 26th of this, but rarely of the plain table.

Having protracted the bounding lines, the particular parts contained within them may be laid down by the proper operations for this purpoſe, delivered in the 26th propoſition; and the method deſcribed in the 27th propoſition may be ſometimes of ſervice; for we may truſt more to the meaſuring of ſides than to the obſerving of angles. We are not to compute four-ſided and many-ſided figures till they are reſolved into triangles, for the ſides do not determine thoſe figures.

In the laying down of cities, or the like, we may make uſe of any of the methods deſcribed above that may be moſt convenient.

The map being finiſhed, it is tranſferred on clean paper by putting the firſt sketch above it, and marking the angles by the point of a ſmall needle. Theſe points being joined by right lines, and the whole illuminated by colours proper to each part, and the figure of the Mariner's compaſs being added to diſtinguiſh the North and South, with a ſcale on the margin, the map or plan will be finiſhed and neat.

We have thus briefly and plainly treated of Surveying, and ſhown by what inſtruments it is performed, having avoided thoſe methods which depend on the magnetick needle, not only becauſe its direction may vary in different places of a field (the contrary of this at leaſt doth not appear) but becauſe the quantity of an angle obſerved by it cannot be exactly known; for an error of two or three degrees can ſcarcely be avoided in taking angles by it. As for the remaining part of ſurveying, whereby the area of a field already laid down on paper is found in acres, roods, or any other ſuperficial meaſures; this we leave to the following part, which treats of the menſuration of ſurfaces.

‘ Beſides the inſtruments deſcribed above, a Surveyor ought to be provided with an off-ſet ſtaff, equal in length to ten links of the chain, and divided into ten equal parts. He ought likewiſe to have ten arrows, or ſmall ſtreight ſticks near two feet long, ſhod with iron ferrils. When the chain is firſt opened, it ought to be examined by the off-ſet ſtaff. In meaſuring any line, the leader of the chain is to have the ten arrows at firſt ſetting out. When the chain is ſtretcht in the line, and the near end touches the place from which you meaſure, the leader ſticks one of the ten arrows in the ground, at the far end of the chain. Then the leader leaving the arrow, proceeds with the chain another length; and the chain being ſtretcht in the line, ſo that the near end touches the firſt arrow, the leader ſticks down another ar-

‘ row at his end of the chain. The line is preſerved ſtreight, if the arrows be always ſet ſo as to be in a right line with the place you meaſure from and that to which you are going. In this manner they proceed till the leader have no more arrows. At the eleventh chain the arrows are to be carried to him again, and he is to ſtick one of them into the ground, at the end of the chain. And the ſame is to be done at the 21, 31, 41, *&c.* chains, if there are ſo many in the right line to be meaſured. In this manner you can hardly commit an error in numbring the chains, unleſs of ten chains at once.

‘ The off-ſet ſtaff ſerves for meaſuring readily the diſtances of any things proper to be repreſented in your plan, from the ſtation-line while you go along. Theſe diſtances ought to be entred into

‘ your field-book, with the corre-‘ ſponding diſtances from the laſt ſta-‘ tion, and proper remarks, that you ‘ may be enabled to plot them juſt-‘ ly, and be in no danger of miſtak-‘ ing one for another, when you ‘ extend your plan. The field-book ‘ may be conveniently divided into ‘ five pages. In the middle column ‘ the angles at the ſeveral ſtations ‘ taken by the Theodolite are to be ‘ entred, with the diſtances from ‘ the ſtations. The diſtances taken ‘ by the off-ſet ſtaff, on either ſide ‘ of the ſtation-line, are to be entred ‘ into columns on either ſide of the ‘ middle-column, according to their ‘ poſition with reſpect to that line. ‘ The names or characters of the ‘ objects, with proper remarks may ‘ be entred in columns on either ‘ ſide of theſe laſt.

‘ Becauſe in the place of the gra-‘ phometer deſcribed by our Author, ‘ ſurveyors now make uſe of the

‘ Theodolite, we ſhall ſubjoin a
‘ deſcription of Mr. *Siſſon*’s lateſt
‘ improved Theodolite from Mr.
‘ *Gardner*’s *practical Surveying improved.*
‘ See a figure of it in plate 4.

‘ In this inſtrument the three
‘ ſtaves by braſs ferrils at top ſcrew
‘ into bell-metal joints, that are
‘ moveable between braſs pillars fixt
‘ in a ſtrong braſs plate, in which
‘ round the center is fixt a ſocket
‘ with a ball moveable in it, and upon
‘ which the four ſcrews preſs, that ſet
‘ the Limb horizontal; next above
‘ is another ſuch plate, thro’ which
‘ the ſaid ſcrews paſs, and on which
‘ round the center is fixt a fruſtum
‘ of a cone of bell-metal, whoſe axis
‘ (being connected with the center
‘ of the ball) is always perpendicular
‘ to the Limb, by means of a conical
‘ braſs ferril fitted to it, whereon
‘ is fixt the Compaſs-box, and on
‘ it the Limb, which is a ſtrong bell-metal
‘ ring, whereon are moveable

three braſs indexes, in whoſe plate are fixt four braſs pillars, that, joining at top, hold the center-pin of the bell-metal double Sextant, whole double Index is fixt on the center of the ſame plate; within the double Sextant is fixt the Spirit-Level, and over it the Teleſcope.

‘The Compaſs-box is graved with two diamonds for *North* and *South*, and with 20 degrees on both ſides of each, that the needle may be ſet to the variation, and its error alſo known.

‘The Limb has two flower-de-luces againſt the diamonds in the Box, inſtead of 180 each, and is curiouſly divided into whole degrees, and number'd to the left hand at every ten to twice 180, having three Indexes diſtant 120, (with *Nonius*'s diviſions on each for the decimals of a degree) that are moved by a pinion fixt below one of them without moving the Limb, and in another is a ſcrew and ſpring under, to fix it to any part of the Limb: it has alſo diviſions number'd for taking the quarter Girt in inches of round *Timber* at the middle height, when ſtanding ten feet horizontally diſtant from its center, which at 20 muſt be doubled, and at 30 trebled, to which a ſhorter index is uſed, having *Nonius*'s diviſions for the decimals of an inch; but an abatement muſt be made for the bark, if not taken off.

‘ The double Sextant is divided on one ſide from under its center (when the Spirit-Tube and Teleſcope are level) to above 60 degrees each way, and numbred at 10, 20, *&c.* and the double Index (through which it is moveable) ſhews on the ſame ſide the degree and decimal of any Altitude or Depreſſion to that extent by *Nonius*'s

‘ diviſions; on the other ſide are ‘ diviſions number'd for taking the ‘ upright height of *Timber*, *&c.* in ‘ feet, when diſtant ten feet, which ‘ at 20 muſt be doubled, and at 30 ‘ trebled; and alſo the quantities ‘ for reducing hypothenuſal Lines ‘ to horizontal: it is moveable by a ‘ pinion fixt in the double index.

‘ The Teleſcope is a little ſhorter ‘ than the diameter of the Limb, ‘ that a fall may not hurt it; yet ‘ it will magnify as much, and ſhew ‘ a diſtant object as perfect, as ‘ moſt of treble its length; in its ‘ focus are very fine croſs-wires, ‘ whoſe interſection is in the plane ‘ of the double Sextant, and this was ‘ a whole Circle, and turned in a ‘ Lathe to a true Plane, and is fixt ‘ at right angles to the Limb; ſo ‘ that whenever the Limb is ſet horizontal (which is readily done by ‘ making the Spirit-Tube level over two ſcrews, and the like over

‘ the other two) the double Sexant ‘ and Teleſcope are moveable in a ‘ vertical Plane, and then every Angle taken on the Limb (tho’ the ‘ Teleſcope be never ſo much elevated or depreſt) will be an Angle ‘ in the Plane of the Horizon, and ‘ this is abſolutely neceſſary in plotting a horizontal Plane.-----

‘ If the Lands to be plotted are ‘ hilly and not in any one Plane, ‘ the Lines meaſured cannot be truly ‘ laid down on paper, without being ‘ reduced to one Plane, which muſt ‘ be the horizontal, becauſe Angles ‘ are taken in that Plane.

‘ In viewing my objects, if they ‘ have much altitude or depreſſion, ‘ I either write down the degree and ‘ decimal ſhewn on the double Sextant, or the links ſhewn on the ‘ back-ſide, which laſt ſubtracted ‘ from every chain in the ſtation-line, leaves the length in the horizontal Plane; but if the degree is

‘ taken, the following Table will ‘ ſhew the quantity.

A Table of the links to be ſubtracted out of every Chain in hypothenuſal Lines *of ſeveral degrees altitude, or depreſſion, for reducing them to* horizontal.

Degrees	*Links*	*Degrees*	*Links*	*Degrees*	*Links*
4,05	¼	14,07	3	23,074	8
5,73	½	16,26	4	24,495	9
7,02	¾	18,195	5	25,84	10
8,11	1	19,95	6	27,13	11
11,48	2	21,565	7	28,36	12

‘ Let the firſt ſtation-line really meaſure 1107 links, and the ‘ Angle of altitude or depreſſion be ‘ 19°,93; looking in the Table I ‘ find againſt 19°,95 is 6 links, ‘ now 6 times 11 is 66, which ſubtracted from 1107 leaves 1041, ‘ the true length to be laid down in ‘ the Plan.

'It is uſeful in ſurveying to take
'the angles which the bounding
'lines form with the magnetick
'needle; in order to check the an-
'gles of the figure, and to plot
'them conveniently afterwards.

PART II.

Of the ſurfaces of bodies.

THE ſmalleſt ſuperficial meaſure with us is a ſquare inch, 144 of which make a ſquare foot. Wrights make uſe of theſe in the meaſuring of deals and planks; but the ſquare foot which the Glaziers in meaſuring of glaſs, conſiſts only of 64 ſquare inches. The other meaſures are, *firſt*, the ell ſquare; 2*dly*, The fall, containing 36 ſquare ells. 3*dly*, The rood containing 40 falls. 4*thly*, The acre containing 4 roods. Slaters, Maſons and Pavers uſe the ell ſquare and the fall. Surveyors of land uſe the ſquare ell, the fall, the rood, and the acre.

The ſuperficial meaſures of the *Engliſh*, are, 1*ſt*, the ſquare foot; 2*dly*, the ſquare yard, containing 9 ſquare feet; for their yard contains only 3 feet; 3*dly*, the pole, containing 30 ¼ ſqaure yards; 4*thly*, the rood, containing 40 poles; 5*thly*, the acre, containing 4 roods. And hence it is eaſy to reduce our ſuperficial meaſures to the *Engliſh*, or theirs to ours.

‘ In order to find the content of
‘ a field, it is moſt convenient to
‘ meaſure the lines by the chains de-
‘ ſcribed above, *p.* 5. that of 22
‘ yards for computing the *Engliſh*
‘ acres, and that of 24 *Scots* ells for
‘ the acres of *Scotland*. The chain
‘ is divided into 100 links, and the

‘ ſquare of the chain is 10000 ſquare links; ten ſquares of the chain, or 100000 ſquare links give an acre. Therefore, if the area be expreſſed by ſquare links, divide by 100000 or cut off five decimal places, and the quotient ſhall give the area in acres and decimals of an acre. Write the entire acres apart, but multiply the decimals of an acre by 4, and the product ſhall give the remainder of the area in roods and decimals of a rood. Let the entire roods be noted apart after the acres, then multiply the decimals of a rood by 40, and the product ſhall give the remainder of the area in falls or poles. Let the entire falls or poles be then writ after the roods, and multiply the decimals of a fall by 36, if the area is required in the meaſures of *Scotland*; but multiply the decimals of a pole by $30\frac{1}{4}$, if the area is required in the meaſures of *Eng-*

‘ *land*, and the product ſhall give ‘ the remainder of the area in ſquare ‘ ells in the former caſe, but in ‘ ſquare yards in the latter. If, in ‘ the former caſe, you would reduce ‘ the decimals of the ſquare ell ‘ to ſquare feet, multiply them by ‘ 9.50694; but in the latter caſe, ‘ the decimals of the *Engliſh* ſquare ‘ yard are reduced to ſquare feet, ‘ by multiplying them by 9.

‘ Suppoſe, for example, that the ‘ area appears to contain 1265842 ‘ ſquare links of the chain of 24 ‘ ells, and that this area is to be expreſſed in acres, roods, falls, &c. ‘ of the meaſures of *Scotland*. Divide the ſquare links by 100000, ‘ and the quotient 12.65842 ſhows ‘ the area to contain 12 acres and ‘ $\frac{65842}{100000}$ of an acre. Multiply the decimal part by 4, and the product ‘ 2.63368 gives the remainder in ‘ roods and decimals of a rood. ‘ Thoſe decimals of the rood being

‘ multiplied by 40, the product
‘ gives 25.3472 falls. Multiply
‘ the decimals of the fall by 36, and
‘ the product gives 12.4992 ſquare
‘ ells. The decimals of the ſquare
‘ ell multiplied by 9.50694 give
‘ 4.7458 ſquare feet. Therefore the
‘ area propoſed amounts to 12 acres,
‘ 2 roods, 25 falls, 12 ſquare ells,
‘ and $4\frac{7458}{10000}$ ſquare feet.

‘ But if the area contains the ſame
‘ number of ſquare links of *Gunter*’s
‘ chain, and is to be expreſſed by
‘ *Engliſh* meaſures, the acres and
‘ roods are computed in the ſame
‘ manner as in the former caſe.
‘ The poles are computed as the
‘ falls. But the decimals of the
‘ pole, *viz.* $\frac{3472}{10000}$ are to be multiplied
‘ by $30\frac{1}{4}$ (or 30.25) and the pro-
‘ duct gives 10.5028 ſquare yards.
‘ The decimals of the ſquare yard
‘ multiplied by 9, give 4.5252 ſquare
‘ feet: therefore in this caſe the a-
‘ rea is in *Engliſh* meaſure 12 acres,

‘ 2 roods, 25 poles, 10 ſquare yards, ‘ and 4 $\frac{5252}{10000}$ ſquare feet.

‘ The *Scots* acre is to the *Engliſh* ‘ acre by ſtatute as 100000 to 78694, ‘ if we have regard to the difference ‘ betwixt the *Scots* and *Engliſh* foot ‘ above mentioned. But it is cuſto‘mary in ſome parts of *England* to ‘ have 18, 21, &c. feet to a pole, ‘ and 160 ſuch poles to an acre; ‘ whereas, by the ſtatute, 16$\frac{1}{2}$ feet ‘ make a pole. In ſuch caſes the ‘ acre is greater in the duplicate ‘ ratio of the number of feet to a ‘ pole.

‘ They who meaſure land in *Scot‘land* by an ell of 37 *Engliſh* inches, ‘ make the acre leſs than the true ‘ *Scots* acre by 593 $\frac{6}{10}$ ſquare *Engliſh* ‘ feet, or by about $\frac{1}{93}$ of the acre.

‘ An husband-land contains 6 acres ‘ of ſock and ſythe land, that is, of ‘ land that may be tilled with a ‘ plough, and mowen with a ſythe; ‘ 13 acres of arable land make an ox-

‘ gang or oxengate; four oxengate make a pound land of old extent (by a decree of the Exchequer, *March* 11. 1585) and is called *librata terræ.* A forty ſhilling land of old extent contains eight oxgang, or 104 acres.

‘ The *arpent* about *Paris* contains 32400 ſquare *Paris* feet, and is equal to $2\frac{2}{3}$ *Scots* roods or $3\frac{37}{100}$ *Engliſh* roods.

‘ The *actus quadratus*, according to *Varro*, *Columella*, &c. was a ſquare of 120 *Roman* feet. The *Jugerum* was the double of this. 'Tis to the *Scots* acre as 10000 to 20456, and to the *Engliſh* acre as 10000 to 16097. It was divided (like the *As*) into 12 *uncias*, and the *uncia* into 24 *ſcrupula*. This with the three preceeding paragraphs are taken from an ingenious manuſcript written by Sir *Robert Stewart* Profeſſor of Natural Philophy. The greateſt part of the table

‘ in *p. 6.* was taken from it likewiſe.

PROPOSITION I.

PROBLEM FIG. 1.

To find the area of a rectangular parallelogram ABCD.

LET the ſide AB, for example, be five feet long, and BC (which conſtitutes with BC a right angle at B) be 17 feet. Let 17 be multiplied by 5, and the product 85 will be the number of ſquare feet in the Area of the figure ABCD. But if the parallelogram propoſed is not rectangular as BEFC, its baſe BC multiplied into its perpendicular height AB (not into its ſide BE) will give its area. This is evident from 35th 1. *Eucl.*

PROPOSITION II.

PROB. FIG. 2.

To find the area of a given triangle.

LET the triangle BAC be given, whoſe baſe BC is ſuppoſed 9 feet long, let the perpendicular AD be drawn from the angle A oppoſite to the baſe, and let us ſuppoſe AD to be 4 feet. Let the half of the perpendicular be multiplied into the baſe, or the half of the baſe into the perpendicular, or take the half of the product of the whole baſe into the perpendicular, the product gives 18 ſquare feet for the area of the given triangle.

But if only the ſides are given, the perpendicular is found either by protracting the triangle or by 12. and 13. 2d *Eucl.* or by trigonometry; but how the area of a triangle may be found from the given ſides only ſhall be ſhown in the 4th Prop. of this part.

PROPOSITION III.

PROB. FIG. 3.

To find the area of any rectilineal figure.

IF the figure be irregular, let it be reſolved into triangles; and drawing perpendiculars to the baſes in each of them, let the area of each triangle be found by the preceeding Prop. and the ſum of theſe areas will give the area of the figure.

SCHOLIUM I.

In meaſuring boards, planks and glaſs, their ſides are to be meaſured by a foot rule divided into 100 equal parts; and after multiplying the ſides, the decimal fractions are eaſily reduced to leſſer denominations. The menſuration of theſe is eaſy, when they are rectangular parallelograms.

SCHOLIUM. 2.

If a field is to be meaſured, let it firſt be plotted on paper by ſome of the methods deſcribed in the preceeding part, and let the figure ſo laid down be divided into triangles, as was ſhown in the preceeding propoſition.

The baſe of any triangle, or the perpendicular upon the baſe, or the diſtance of any two points of the field is meaſured, by applying it to the ſcale according to which the map is drawn.

SCHOLIUM 3.

But if the field given be not in a horizontal plain, but uneven and mountainous, the ſcale gives the horizontal line between any two points, but not their diſtance meaſured on the uneven ſurface of the field. And indeed it would appear that the horizontal plain is to be accounted the area of an uneven and hilly country. For if ſuch ground is laid out for building on, or for planting with trees or bearing corn, ſince theſe ſtand perpendicular to the horizon, it is plain that a mountainous country cannot be conſidered as of greater extent for thoſe uſes than the horizontal plain; nay, perhaps, for nouriſhing of plants, the horizontal plain may be preferable.

If however the area of a figure as it lies irregularly on the ſurface of the earth, is to be meaſured, this may be eaſily done by reſolving it into triangles as it lyes. The ſum of their areas will be the area ſought, which exceeds the area of the horizontal figure more or leſs according as the field is more or leſs uneven.

PRO-

PROPOSITION IV.

PROB. FIG. 2.

The ſides of a triangle being given, to find the area, without finding the perpendicular.

LET all the ſides of the triangle be collected into one ſum, from the half of which let the ſides be ſeparately ſubſtracted, that three differences may be found betwixt the foreſaid half ſum and each ſide; then let theſe three differences, and the half ſum be multiplied into one another, and the ſquare-root of the product will give the area of the triangle. For example, let the ſides be 10, 17, 21, the half of their ſum is 24, the three differences betwixt this half ſum and the three ſides, are 14, 7 and 3. The firſt being multiplied by the ſecond, and their product by the third, we have 294 for the product of the differences, which multiplied by the foreſaid half ſum 24 gives 7056; the ſquare-root of which 84 is the area of the triangle. The demonſtration of this, for the ſake of brevity we omit. It is to be found in ſeveral treatiſes, particularly in *Clavius*'s *practical Geometry*.

PROPOSITION V.

THEOR. FIG. 4.

The area of the ordinate figure ABEFGH is equal to the product of the half-circumference of the polygon multiplied into the perpendicular drawn from the center of the circumſcribed circle to the ſide of the polygon.

FOr the ordinate figure can be reſolved into as many equal triangles, as there are ſides of the figure;

and ſince each triangle is equal to the product of half the baſe into the perpendicular, it is evident that the ſum of all the triangles together, that is the polygon, is equal to the product of half the ſum of the baſes (that is the half of the circumference of the polygon) into the common perpendicular height of the triangles drawn from the center C to one of the ſides, for example to AB.

PROPOSITION VI.

Prob. Fig. 5.

The area of a circle is found by multiplying the half of the periphery into the radius, or the half of the radius into the periphery.

FOR a circle is not different from an ordinate or regular polygon of an infinite number of ſides, and the common height of the triangles into which the polygon (or circle) may be ſuppoſed to be divided, is the radius of the circle.

Were it worth while, it were eaſy to demonſtrate accurately this propoſition by means of the inſcribed and circumſcribed figures, as is done in the 5th prop. of the treatiſe of *Archimedes*, concerning the dimenſions of the circle.

COROLLARY.

Hence alſo it appears that the area of the ſector ABCD is produced by multiplying the half of the arch into the radius; and likewiſe that the area of the ſegment of the circle ADC is found by ſubſtracting from the area of the ſector the area of the triangle ABC.

PROPOSITION VII.

THEOR. FIG. 6.

The circle is to the ſquare of the diameter as 11 *to* 14 *nearly.*

FOR if the diameter AB be ſuppoſed to be 7, the circumference AHKB will be almoſt 22 (by the 22d prop. of the firſt part of this) and the area of the ſquare DC will be 49; and by the preceeding prop. of this, the area of the circle will be 38 $\frac{1}{2}$, therefore the ſquare DC will be to the inſcribed circle as 49 to 38$\frac{1}{2}$, or as 98 to 77, that is, as 14 to 11. *Q. E. D.*

If greater exactneſs is required, you may proceed to any degree of accuracy; for the ſquare DC is to the inſcribed circle as 1 to $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13}$ *&c. in infinitum.*

‘ This ſeries will be of no ſervice

‘ for computing the area of the circle accurately, without ſome further artifice, becauſe it converges at too ſlow a rate. The area of the circle will be found exactly enough for moſt purpoſes, by multiplying the ſquare of the diameter by 7854, and dividing by 10000, or cutting off four decimal places from the product; for the area of the circle is to the circumſcribed ſquare nearly as 7854 to 10000.

PROPOSITION VIII.

PROB. FIG. 7.

To find the area of a given ellipſe.

LEt ABCD be an ellipſe, whoſe greater diameter is BD and leſſer AC, biſecting the greater perpendicularly in E. Let a mean proportional HF be found (by the 13th

6. *Eucl.*) between AC and BD, and (by the 6th of this) find the area of the circle deſcribed on the diameter HF. I ſay that this area is equal to the area of the ellipſe ABCD. For becauſe, as BD to AC, ſo the ſquare of BD to the ſquare of HF (by 2. *Cor.* 20th 6. *Eucl.*) but (by the 2d 12. *Eucl.*) as the ſquare of BD to the ſquare of HF, ſo is the circle of the diameter BD to the of the diameter HF: therefore as BD to AC, ſo the circle of the diameter BD to the circle of the diameter HF. And (by the 5. prop. of *Archimedes* of ſpheroids) as the greater diameter BD to the leſſer AC, ſo is the circle of the diameter BD to the ellipſe ABCD. Conſequently (by the 11th 5. *Eucl.*) the cirlec of the diameter BD will have the ſame proportion to the circle of the diameter HF, and to the ellipſe ABCD. Therefore, by 9th 5. *Eucl.*

the area of the circle of the diameter HF will be equal to the area of the ellipſe ABCD. *Q. E. D.*

SCHOLIUM.

From this and the two preceeding propoſitions, a method is derived of finding the area of an ellipſe. There are two ways; 1ſt, ſay, as 1 is to the leſſer diameter, ſo is the greater diameter to a fourth number (which is found by the rule of three.) Then again ſay, as 14 to 11, ſo is the 4th number found to the area ſought. But the ſecond way is ſhorter. Multiply the leſſer diameter into the greater, and the product by 11; then divide the whole product by 14, and the quotient will be the area ſought of the ellipſe. For example, let the greater diameter be 10, and the leſſer 7, by multiplying 10 by 7, the product is 70, and multiplying that by 11, it is 770, and dividing 770 by 14, the quotient will be 55, which is the area of the ellipſis ſought.

‘ The area of the ellipſe will be ‘ found more accurately, by multi- ‘ plying the product of the two di- ‘ ameters by .7854.

We ſhall add no more about other plain ſurfaces, whether rectilinear or curvilinear, which ſeldom occur in practice; but ſhall ſubjoin ſome propoſitions about meaſuring the ſurfaces of ſolids.

PROPOSITION IX. PROB.

To meaſure the ſurface of any priſm.

BY the 14th definition of the 11. *Eucl.* a priſm is contained by plains, of which two oppoſite ſides (commonly called the baſes) are plain rectilinear figures; which are either regular and ordinate, and meaſured by prop. 5. of this part, or however irregular, and then they are meaſured by the 3d prop. of this book. The other ſides are parallelograms which are meaſured by the 1. prop. of this ſecond part, and the whole ſuperficies of the priſm, conſiſts of the ſum of thoſe taken altogether.

PROPOSITION X. PROB.

To meaſure the ſuperficies of any pyramid.

SInce its baſis is a rectilinear figure, and the reſt of the plains terminating in the top of the pyramid are triangles, theſe meaſured ſeparately, and added together, give the ſurface of the pyramid required.

PRO-

PROPOSITION XI. PROB.

To meaſure the ſuperficies of any regular body.

THeſe bodies are called regular, which are bounded by equilateral and equiangular figures. The ſuperficies of the tetraedron conſiſts of four equal and equiangular triangles; the ſuperficies of a hexaedron or cube, of ſix equal ſquares; an octaedron of eight equal equilateral triangles; a dodecaedron of twelve equal and ordinate pentagons. And the ſuperficies of an icoſiaedron of twenty equal and equilateral triangles. Therefore it will be eaſy to meaſure theſe ſurfaces from what has been already ſhown.

In the ſame manner we may meaſure the ſuperficies of a ſolid contained by any plains.

PROPOSITION XII.

PROB. FIG. 8.

To meaſure the ſuperficies of a cylinder.

BEcauſe a cylinder differs very little from a priſm whoſe oppoſite plains (or baſes) are ordinate figures of an infinite number of ſides, it appears that the ſuperficies of a cylinder, without the baſes, is equal to an infinite number of parallelograms, the common altitude of all which is, with the height of the cylinder, and the baſes of them all differ very little from the periphery of the circle which is the baſe of the cylinder. Therefore this periphery multiplied into the common height, gives the ſuperficies of the cylinder, excluding the baſes; which are to be meaſured ſeparately by the help of the 6th prop. of this part.

This propoſition concerning the meaſure of the ſurface of the cylinder (excluding its baſis) is evident from this, That when it is conceived to be ſpread out, it becomes a parallelogram, whoſe baſe is the periphery of the circle of the baſe of the cylinder ſtretched into a right line, and whoſe height is the ſame with the height of the cylinder.

PROPOSITION XIII.

PROB. FIG. 9.

To meaſure the ſurface of a right cone.

THe ſurface of a right cone is very little different from the ſurface of a right pyramid, having an ordinate polygon for its baſe of an infinite number of ſides; the ſurface of which (excluding the baſe) is equal to the ſum of the triangles.

The ſum of the baſes of theſe triangles is equal to the periphery of the circle of the baſe, and the common height of the triangles is the ſide of the cone AB; wherefore the ſum of theſe triangles is equal to the product of the ſum of the baſes (*i. e.* the periphery of the baſe of the cone) multiplied into the half of the common height, or it is equal to the product of the ſide of the cone multiplied into the half of the periphery of the baſe.

If the area of the baſe is likewiſe wanted, it is to be found ſeparately by the 6th prop. of this part. If the ſurface of a cone is ſuppoſed to be ſpread out on a plane, it will become a ſector of a circle, whoſe radius is the ſide of the cone, and the arch terminating the ſector is made from the periphery of the baſe. Whence by corol. 6. prop. of this, its dimenſion may be found.

CO-

COROLLARY.

Hence it will be eaſy to meaſure the ſurface of a *fruſtum* of a cone cut by a plain parallel to the baſe. As to what relates to the meaſuring of the ſurface of the ſcalenous cone, becauſe it is not very uſeful in practice, we ſhall not deſcribe the method, which would carry us beyond the limits of this treatiſe.

PROPOSITION XIV.

PROB. FIG. 10.

To meaſure the ſurface of a given ſphere.

LEt there be a ſphere, whoſe center is A, and let the area of its convex ſurface be required. *Archimedes* demonſtrates (37. prop. 1. book of the ſphere and cylinder) that its ſurface is equal to the area of four great circles of the ſphere; that is, let the area of the great circle be multiplied by 4, and the product will give the area of the ſphere, or by 20th 6. and 2d 12. of *Eucl.* the area of the ſphere given is equal to the area of a circle whoſe radius is the right line BC the diameter of the ſphere. Therefore having meaſured (by 6. prop. of this part) the circle deſcribed with the radius BC, this will give the ſurface of the ſphere.

PROPOSITION XV.

PROB. FIG. 10.

To meaſure the ſurface of a ſegment of a ſphere.

LET there be a ſegment cut off by the plain ED. *Archimedes* demonſtrates (49. and 50. 1. *de ſphæra*) that the ſurface of this ſegment, exclucluding the circular baſe, is equal to the area of a circle whoſe radius is the right line BE drawn from the vertex B of the ſegment to the periphery of the circle DE. Therefore by the 6. prop. of this part, it is eaſily meaſured.

COROLLARY 1.

Hence that part of the ſurface of a ſphere that lieth between two parallel plains is eaſily meaſured, by ſubſtracting the ſurface of the leſſer ſegment from the ſurface of the greater ſegment.

COROLLARY 2.

Hence likewiſe it follows that the ſurface of a cylinder deſcribed about a ſphere (excluding the baſis) is equal to the ſurface of the ſphere, and the parts of the one to the parts of the other intercepted between plains pa-

parallel to the baſis of the cylinder.

PART III.

Of ſolid figures and their menſuration.

AS in the preceeding parts we took an inch for the ſmalleſt meaſure in length, and an inch ſquare for the ſmalleſt ſuperficial meaſure; ſo now, in treating of the menſuration of ſolids, we take a cubical inch for the ſmalleſt ſolid meaſure. Of theſe 109 make a *Scots* pint; other liquid meaſures depend on this, as is generally known.

In dry meaſures, the firlot by ſtatute, contains 19$\frac{1}{2}$ pints, and on this depend the other dry meaſures: thereler.

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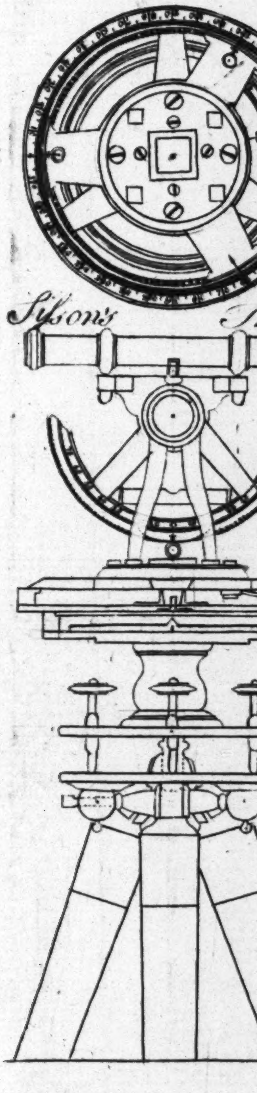
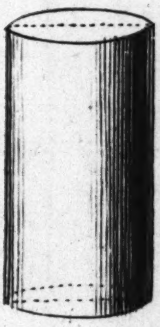


Siſson's

FIG. 8.



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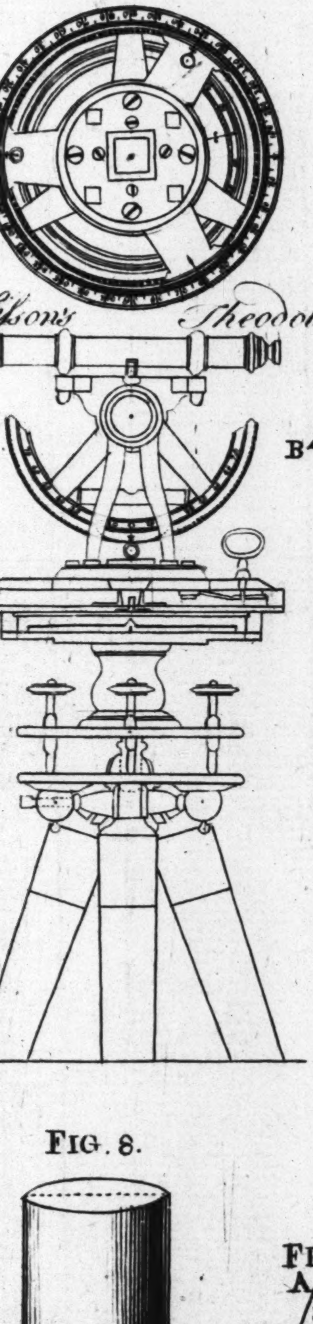

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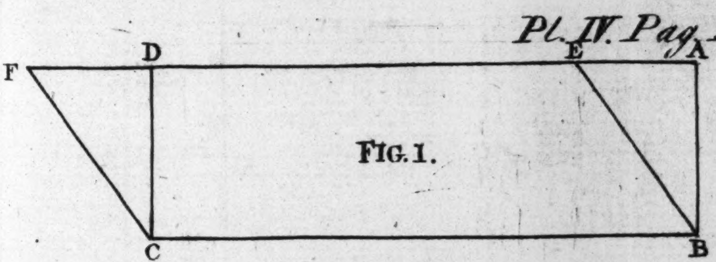


FIG. 1.

FIG. 2.

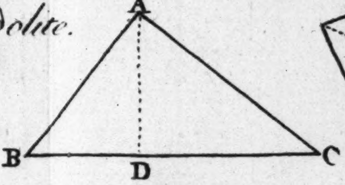


FIG. 3.

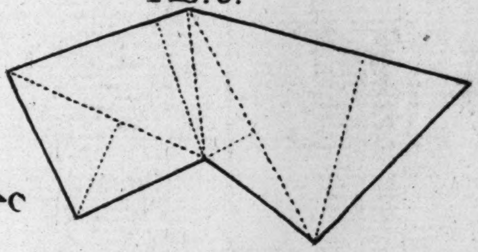


FIG. 4.

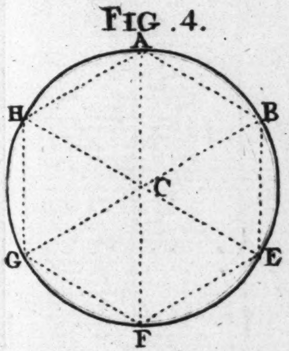


FIG. 5.

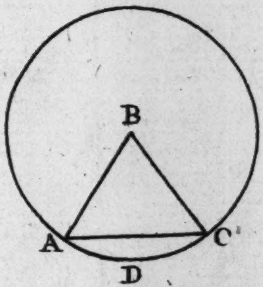


FIG 6.

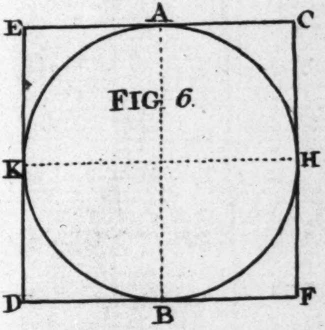


FIG. 7.

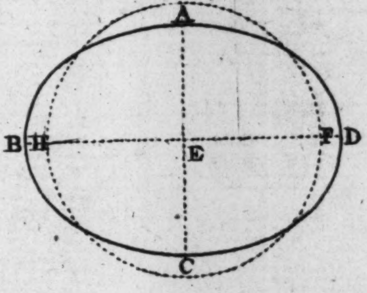


FIG. 8.

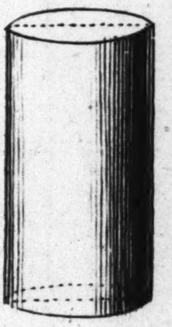


FIG. 9.

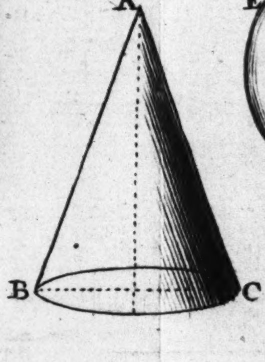


FIG. 10.

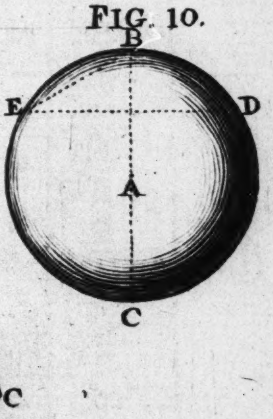
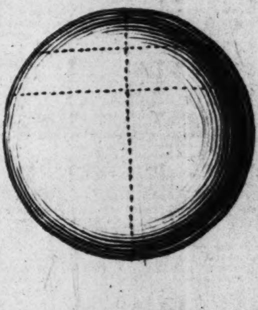


FIG. 11.



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therefore, if the content of any ſolid be given in cubical inches, it will be eaſy to reduce the ſame to the common liquid or dry meaſures, and converſely to reduce theſe to ſolid inches. The liquid and dry meaſures in uſe among other nations are known from their writers.

‘ As to the *Engliſh* liquid mea-
‘ ſures, By act of Parliament 1706,
‘ any round veſſel, commonly cal-
‘ led a cylinder, having an even
‘ bottom, being ſeven inches in di-
‘ ameter throughout, and ſix inches
‘ deep from the top of the inſide
‘ to the bottom; (which veſſel will
‘ be found by computation to con-
‘ tain 230 $\frac{907}{1000}$ cubical inches) or a-
‘ ny veſſel containing 231 cubical
‘ inches, and no more, is deemed to
‘ to be a lawful wine gallon. An
‘ *Engliſh* pint therefore contains
‘ 28 $\frac{7}{8}$ cubical inches; two pints make
‘ a quart, four quarts a gallon, 18
‘ gallons a roundlet, three round-

lets and an half, or 63 gallons make a hogſhead; the half of a hogſhead is a barrel; one hogſhead, and a third, or 84 gallons make a puncheon; one puncheon and a half, or two hogſheads, or 126 gallons, make a pipe or butt; the third part of a pipe, or 42 gallons make a tierce; two pipes, or three puncheons, or four hogſheads make a tun of wine. Tho' the *Engliſh* wine-gallon is now fixed at 231 cubical inches, the ſtandard kept in *Guildhall* being meaſured before many perſons of diſtinction, *May* 25. 1688, it was found to contain only 224 ſuch inches.

‘ In the *Engliſh* Beer-meaſure, a gallon contains 282 cubical inches; conſequently 35 $\frac{1}{4}$ cubical inches make a pint, two pints make a quart, four quarts make a gallon, nine gallons a firkin, four firkins a barrel. In ale, eight gallons make

‘ a firkin, and 32 gallons make a ‘ a barrel. By an Act of the firſt ‘ of *William* and *Mary*, 34 gallons ‘ is the barrel, both for beer and ‘ ale, in all places; except within ‘ the weekly bills of mortality.

‘ In *Scotland* it is known that four ‘ gills make a mutchkin, two mutchkins make a chopin, a pint is two ‘ chopins, a quart is two pints, and ‘ a gallon is four quarts or eight ‘ pints. The accounts of the cubical inches contained in the *Scots* ‘ pint vary conſiderably from each ‘ other. According to our Author ‘ it contains 109 cubical inches. ‘ But the ſtandard jugs kept by the ‘ Dean of Gild of *Edinburgh* (one ‘ of which has the year 1555, with ‘ the arms of *Scotland*, and of the ‘ town of *Edinburgh* marked upon it) ‘ having been carefully meaſured ſeveral times, and by different perſons, the *Scots* pint, according ‘ to thoſe ſtandards, was found to

‘ contain about $103\frac{4}{10}$ cubic inches. The Pewterers jugs (by which the veſſels in common uſe are made) are ſaid to contain ſometimes betwixt 105 and 106 cubic inches. A cask that was meaſured by the Brewers of *Edinburgh*, before the Commiſſioners of Exciſe in 1707, was found to contain $46\frac{7}{8}$ *Scots* pints; the ſame veſſel contained $18\frac{1}{16}$ *Engliſh* ale gallons. Suppoſing this menſuration to be juſt, the *Scots* pint will be to the *Engliſh* ale gallon as 289 to 750; and if the *Engliſh* ale-gallon be ſuppoſed to contain 282 cubical inches, the *Scots* pint will contain 108.664 cubical inches. But it is ſuſpected on ſeveral grounds that this experiment was not made with ſufficient care and exactneſs.

‘ The Commiſſioners appointed by authority of Parliament to ſettle the meaſures and weights in their Act of *February* 19. 1618, relate,

‘ That having cauſed fill the *Linlithgow* firlot with water, they found that it contained 21 $\frac{1}{4}$ pints of the juſt *Stirling* jug and meaſure. They likewiſe ordain that this ſhall be the juſt and only firlot, and add, *That the wideneſs and breadneſs of the which firlot, under and above, even over within the buirds, ſhall contain nineteen inches, and the ſixth part of an inch, and the deipneſs ſeven inches, and a third part of an inch.* According to this Act (ſuppoſing their experiment and computation to have been accurate) the pint contained only 99.56 cubical inches; for the content of ſuch a veſſel as is deſcribed in the act is 2115.85, and this divided by 21 $\frac{1}{4}$, gives 99.56. But by the weight of water ſaid to fill this firlot in the ſame Act, the meaſure of the pint agrees nearly with the *Edinburgh* ſtandard above mentioned.

‘ As for the *Engliſh* meaſures of

corn the *Wincheſter* gallon contains 272 ¼ cubical inches, two gallons make a peck, four pecks or eight gallons (that is 2178 cubical inches) make a buſhel, and a quarter is eight buſhels.

‘ Our Author ſays, that 19 ½ *Scots* pints, make a firlot; but this does not appear to be agreeable to the ſtatute above mentioned, nor to the ſtandard jugs. It may be conjectured that the proportion aſſigned by him has been deduced from ſome experiment of how many pints, according to common uſe, were contained in the firlot. For if we ſuppoſe thoſe pints to have been each of 108.664 cubical inches, according to the experiment made in the 1707, before the Commiſſioners of Exciſe deſcribed above; then 19 ½ ſuch pints will amount to 2118.94 cubical inches which agrees nearly with 2115.85 the meaſure of the firlot by the ſtatute

‘ above mentioned. But it is probable, that in this he followed the Act 1587, where it is ordained that the wheat firlot ſhall contain 19 pints and twa joucattes. A wheat firlot marked with the *Linlithgow* ſtamps being meaſured, was found to contain about 2211 cubical inches. By the ſtatute of 1618, the barley-firlot was to contain 31 pints of the juſt *Stirling* jug.

‘ A *Paris* pint is 48 cubical *Paris* inches, and is nearly equal to an *Engliſh* wine quart. The *Boiſſeau* contains 644.68099 *Paris* cubical inches, or 780.36 *Engliſh* cubical inches.

‘ The *Roman Amphora* was a cubical *Roman* foot, the *Congius* was the 8th part of the *Amphora*, the *Sextarius* was one ſixth of the *Congius*. They divided the *Sextarius* like the *As* or *Libra*. Of dry meaſures the *Medimnus* was equal to two *Amphoras*, that is about 1 $\frac{11}{27}$

‘ *Engliſh* legal buſhels; and the *Me-*
‘ *dius* was the third part of the *Am-*
‘ *phora.*

PROPOSITION I. PROB.

To find the ſolid content of a given priſm.

BY the 2. prop. of the 2d part of this, let the area of the baſe of the priſm be meaſured, and be multiplied by the height of the priſm, the product will give the ſolid content of the priſm.

PROPOSITION II. PROB.

To find the ſolid content of a given pyramid.

THE area of the baſe being found (by the 3d prop. of the 2d part) let it be multiplied by the third part of the height of the pyramid, or the third part of the baſe by the height, the product will give the ſolid content by 7th 12. *Eucl.*

CORROLARY.

If the ſolid content of a *fruſtum* of a pyramid is required, firſt let the ſolid content of the entire pyramid be found, from which ſubſtract the ſolid content of the part that is wanting, and the ſolid content of the broken pyramid will remain.

PROPOSITION III. Prob.

To find the content of a given cylinder.

THE area of the baſe being found (by prop. 6. of 2. part) if it be a circle, and by prop. 8. if it be an ellipſe for in both caſes it is a cylinder) multiply it by the height of the cylinder, and the ſolid content of the cylinder will be produced.

CO-

COROLLARY, Fig. 1.

And in this manner may be meaſured the ſolid content of veſſels and casks not much different from a cylinder as ABCD. If towards the middle EF it be ſomewhat groſſer, the area of the circle of the baſe being found (by 6th prop. of 2. part) and added to the area of the middle circle EF, and the half of their ſum (that is an arithmetical mean between the area of the baſe, and the area of the middle circle) taken for the baſe of the veſſel, and multiplied into its height, the ſolid content of the given veſſel will be produced.

Note, That the length of the veſſel as well as the diameters of the baſe and of the circle EF ought to be taken within the ſtaves; for it is the ſolid content within the ſtaves that is ſought.

PRO-

PROPOSITION IV. PROB.

To find the ſolid content of a given cone.

LET the area of the baſe (found by prop. 6th, 2. part) be multiplied into $\frac{1}{3}$ of the height, the product ſhall give the ſolid content of the cone; for (by 10th, 12. *Eucl.*) a cone is the third part of a cylinder that has the ſame baſe and height.

PROPOSITION V.

PROB. FIG. 2. and 3.

To find the ſolid content of a fruſtum *of a cone cut by a plane parallel to the plain of the baſe.*

FIrſt, let the height of the entire cone be found, and thence, (by the preceeding prop.) its ſolid content; from which ſubſtract the ſolid content of the cone cut off at the top, there will remain the ſolid content of the *fruſtum* of the cone.

How the content of the entire cone may be found, appears thus, let ABCD be the *fruſtum* of the cone (either right or ſcalenous, as in the figures 2. and 3.) let the cone ECD be ſuppoſed to be compleated; let AG be drawn parallel to DE, and let AH and EF be perpendicular on CD. It will be (by 2d 6. *Eucl.*) as CG : CA : : CD : CE; but (by the 4th prop. of the ſame book) as CA : AH : : CE : EF; conſequently (by 22d 5. *Eucl.*) as CG : AH : : CD : EF, that is, as the exceſs of the diameter of the greater baſe of the *fruſtum* above the leſſer baſe, is to the height of the *fruſtum*, ſo is the diameter of the greater baſe to the height of the entire cone.

COROLLARY, FIG. 4.

Some casks whoſe ſtaves are remarkably bended about the middle, and ſtreight towards the ends, may be taken for two portions of cones, without any conſiderable error. Thus ABEF is a *fruſtum* of a right cone, to whoſe baſe EF, on the other ſide, there is another ſimilar *fruſtum* of a cone joined EDCF. The vertices of theſe cones, if they be ſuppoſed to be completed, will be found at G and H. Whence, by the preceeding prop. the ſolid content of ſuch veſſels may be found.

PROPOSITION VI.

THEOR. FIG. 5.

A Cylinder circumſcribed about a ſphere, that is, having its baſe equal to a great circle of the ſphere, and its height equal to the diameter of the ſphere, is to the ſphere as 3 to 2.

Let ABEC be the quadrant of a circle, and ABDC the circumſcribed ſquare; and likewiſe the triangle ADC; by the revolution of the figure about the right line AC as axis, a hemiſphere will be generated by the quadrant, a cylinder of the ſame baſe and height by the ſquare, and a cone by the triangle. Let theſe three be cut any how by the plain HF parallel to the baſe AB, the ſection in the cylinder will be a a circle whoſe radius is FH; in the hemiſphere a circle of the radius EF; and in the cone, a circle of the radius GF.

By the 47th 1. *Eucl.* EAq, or HFq = EFq and FAq taken together, (but AFq = FGq, becauſe AC = CD) therefore the circle of the radius HF is equal to a circle of the radius EF together with a circle of the radius

GF, and ſince this is true every where, all the circles together deſcribed by the reſpective *radii* HF, (that is the cylinder,) are equal to all the circles deſcribed by the reſpective *radii* EF and FG (that is to the hemiſphere and the cone taken together;) but by 10th 12. *Eucl.* the cone generated by the triangle DAC is one third part of the cylinder generated by the ſquare BC. Whence it follows that the hemiſphere generated by the rotation of the quadrant ABEC, is equal to the remaining two third parts of the cylinder, and that the whole ſphere is $\frac{2}{3}$ of the double cylinder circumſcribed about it.

This is that celebrated 39th prop. 1. book of *Archimedes* of the ſphere and cylinder, in which he determines the proportion of the cylinder to the ſphere inſcribed to be that of 3 to 2.

COROLLARY.

Hence it follows that the ſphere is equal to a cone having a circle equal to the ſuperficies of the ſphere, or to four great circles of the ſphere, or to a circle whoſe radius is equal to the diameter of the ſphere (by 14th prop. 2. part of this) for its baſe, and its height equal to the ſemidiameter of the ſphere. And indeed a ſphere differs very little from the ſum of an infinite number of cones, that have their baſes in the ſurface of the ſphere, and their common vertex in the center of the ſphere: ſo that the ſuperficies of the ſphere, (of whoſe dimenſion ſee 14th prop. 2. part of this) multiplied into the third part of the ſemidiameter, gives the ſolid content of the ſphere,

PROPOSITION VII.

PROB. FIG. 6.

To find the ſolid content of a ſector of the ſphere.

A Spherical ſector ABC (as appears by the corol. of the preceeding prop.) is very little different from an infinite number of cones, having their baſes in the ſuperficies of the ſphere BEC, and their common vertex in the center. Wherefore the ſpherical ſuperficies BEC being found (by 15. prop. 2. part) and multiplied into the third part of AB the radius of the ſphere, the product will give the ſolid content of the ſector ABC.

COROLLARY.

It is evident how to find the ſolidity of a ſpherical ſegment leſs than a hemiſphere, by ſubſtracting the cone ABC from the ſector already found. But if the ſpherical ſegment be greater than a hemiſphere, the cone correſponding muſt be added to the ſector, to make the ſegment.

PROPOSITION VIII.

PROB. FIG. 7.

To find the ſolidity of the ſpheroid, and of its ſegments cut by plains perpendicular to the axis.

IN the ſecond prop. of this part, it is ſhown that every where EH : EG : : CF : CD ; but circles are as the ſquares deſcribed upon their rays, that is, the circle of the radius EH, is to the circle of the radius EG as CFq to CDq. And ſince it is ſo every where, all the circles deſcribed with the reſpective rays EH (that is the ſpheroid made by the rotation of the ſemiellipſis AFB around the axis AB) will be to all the circles deſcribed by the reſpective *radii* EG (that is the ſphere deſcribed by the rotation of the ſemicircle ADB on the axis AB) as FG*q* to CD*q*; that is, as the ſpheroid to the ſphere on the ſame axis, ſo is the ſquare of the other axis of the generating ellipſe to the ſquare of the axis of the ſphere.

And this holds, whether the ſpheroid be found by a revolution around the greater or leſſer axis.

COROLLARY 1.

Hence it appears that the half of the ſpheroid, formed by the rotation of the ſpace AHFC around the axis AC, is double of the cone generated by the triangle AFC about the ſame axis; which is the 32d prop. of *Archimedes*, of conoids and ſpheroids.

COROLLARY 2.

Hence likewiſe is evident the meaſure of ſegments of the ſpheroid cut by plains perpendicular to the axis. For the ſegment of the ſpheroid made by the rotation of the ſpace ANHE round the axis AE, is to the ſegment of the ſphere having the ſame axis AC, and made by the rotation of the ſegment of the circle AMGE, as CFq to CDq.

But if the meaſure of this ſolid be wanted with leſs labour, by the 34th prop. of *Archimedes*, of conoids and ſpheroids, it will be as BE to AC + EB, ſo is the cone of the ſame baſe and height to the ſegment of the ſphere made by the rotation of the ſpace ANHE around the axis EA; which could eaſily be demonſtrated, (was this a proper place for it) by the method of indiviſibles.

COROLLARY 3.

Hence it is eaſy to find the ſolid content of the ſegment of a ſphere or ſpheroid intercepted between two parallel plains, perpendicular to the axis. This agrees as well to the oblate as to the oblong ſpheroid, as is obvious.

COROLLARY 4. FIG. 8.

If a cask is to be valued as the middle piece of an oblong ſpheroid, cut by the two plains DC and FG, at right angles to the axis. Firſt let the ſolid content of the half ſpheroid ABCED be meaſured by the preceeding prop. from which let the ſolidity of the ſegment DEC be ſubſtracted, and there will remain the ſegment ABCD, and this doubled, will give the capacity of the cask required.

The following method is generally made uſe of for finding the ſolid content of ſuch veſſels. The double area of the greateſt circle, that is of that which is deſcribed by the diameter AB at the middle of the cask, is added to the area of the circle at the end, that is, of the circle DC or FG (for they are uſually equal) and the third part of this ſum is taken for a mean baſe of the cask, which therefore multiplied into the length of the cask OP gives the content of the veſſel required.

Sometimes veſſels have other figures, different from thoſe we have mentioned; the eaſy methods of meaſuring which may be learned from thoſe who practiſe this art. What hath already been delivered is ſufficient for our purpoſe.

PROPOSITION IX.

Prob. Fig. 9. and 10.

To find how much is contained in a veſſel that is in part empty, whoſe axis is parallel to the horizon.

LET AGBH be the great circle in the middle of the cask, whoſe ſegment GBH is filled with liquor, the ſegment GAH being empty; the ſegment GBH is known, if the depth EB be known, and EH a mean proportional between the ſegments of the diameter AE and EB; which are found by a rod or ruler put into the veſſel at the orifice. Let the baſis of the cask, at a medium, be found, which ſuppoſe to be the circle CKDL, and let the ſegment KCL be ſimilar to the ſegment GAH (which is either found by the rule of three, becauſe as the circle AGBH is to the circle CKDL, ſo is the ſegment GAH to the ſegment KCL; or is found from the tables of ſegments made by authors) and the product of this ſegment multiplied by the length of the cask, will give the liquid content remaining in the cask.

PROPOSITION X. PROB.

To find the ſolid content of a regular and ordinate body.

A Tetraedron being a pyramid, the ſolid content is found by the ſecond prop. of this part. The hexaedron or cube, being a kind of priſm, it is meaſured by the firſt prop. of this part. An octaedron conſiſts of two pyramids of the ſame ſquare baſe, and of equal heights, conſequently its meaſure is found from the ſecond prop. of this part. A dodecaedron conſiſts of twelve pyramids having equal equilateral and equiangular pentagonal baſes; and ſo one of theſe being meaſured (by 2d prop. of this) and multiplied by 12, the product will be equal to the ſolid content of the dodecaedron. The icoſiaedron conſiſts of 20 equal pyramids having triangular baſes, the ſolid content of one of which being found (by the 2d prop. of this) and multiplied by 20, gives the whole ſolid. The baſes and heights of theſe pyramids, if you want to proceed more exactly, may be found by Trigonometry.

PROPOSITION XI. PROB.

To find the ſolid content of a body however irregular.

LET the given body be immerſed in a veſſel of water, having the figure of a parallellopipedon or priſm, and let it be noted how much the water is raiſed upon the immerſion of the body. For it is plain that the ſpace which the water fills, after the immerſion of the body, exceeds the ſpace filled before its immerſion, by a ſpace equal to the ſolid content of the body however irregular. But when this exceſs is of the figure of a parallellopipedon or priſm, it is eaſily meaſured by the firſt prop. of this part, to wit, by multiplying the area of the baſe, or mouth of the veſſel, into the difference of the elevations of the water before and after immerſion. Whence is found the ſolid content of the body given. *Q. E. I.*

In the ſame way the ſolid content of a part of a body may be found, by immerſing that part only in water.

There is no neceſſity to inſiſt here on diminiſhing or enlarging ſolid bodies in a given proportion. It will be eaſy to deduce theſe things from the 11th and 12th B. of *Euclid.*

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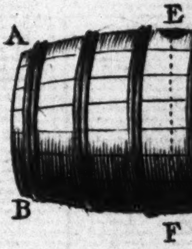

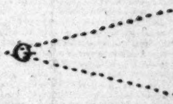

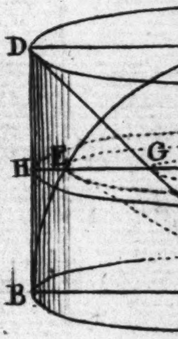

FIG. 6.

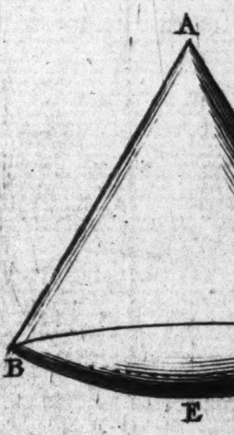

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FIG. 1.

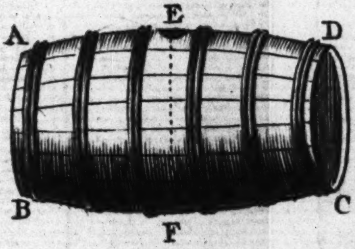


FIG. 2.

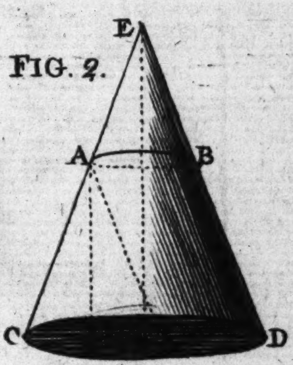


FIG. 3.

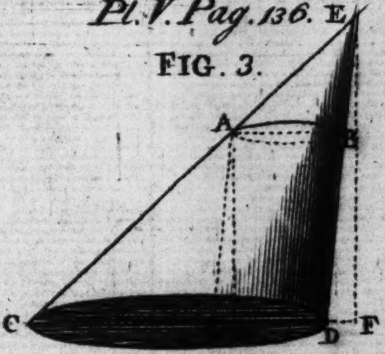


FIG. 4.

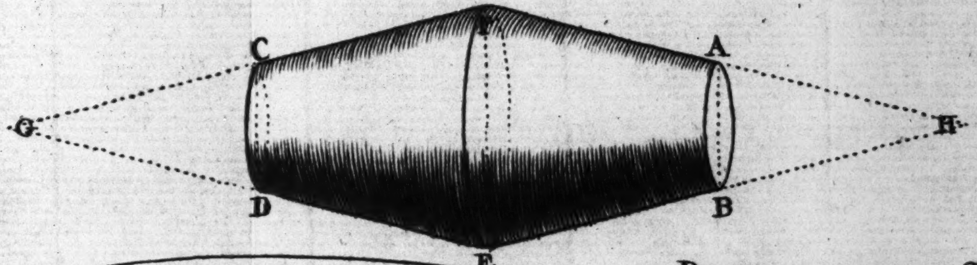


FIG. 5.

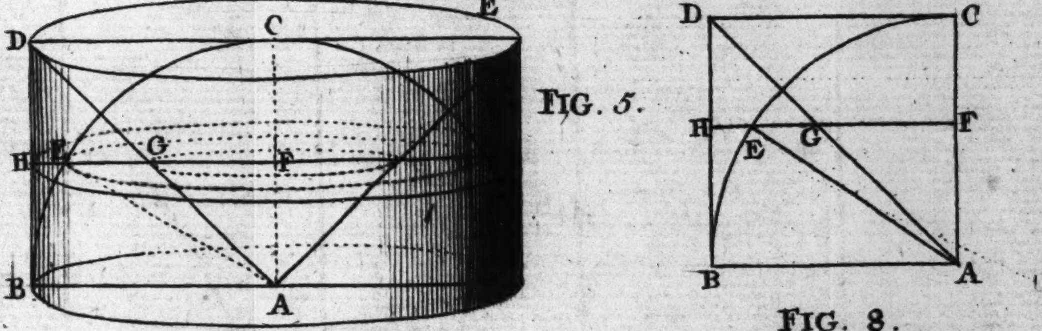


FIG. 7.

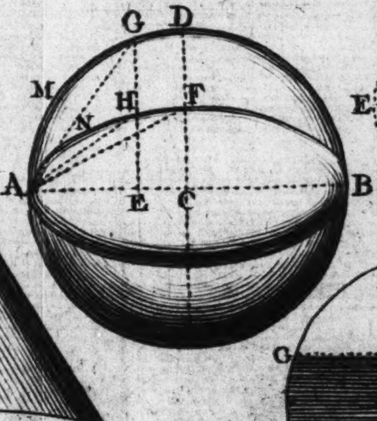


FIG. 6.

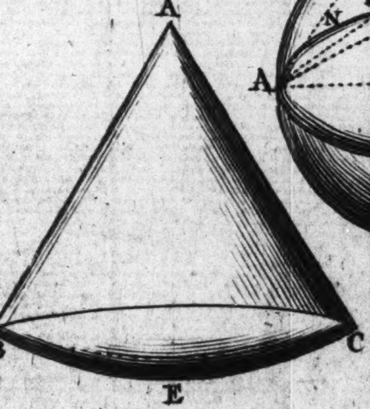


FIG. 8.

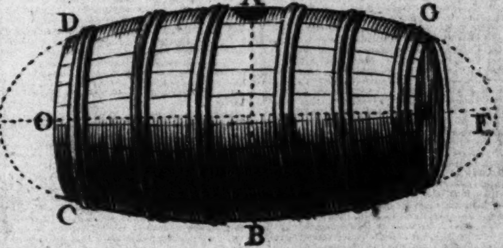


FIG. 9.

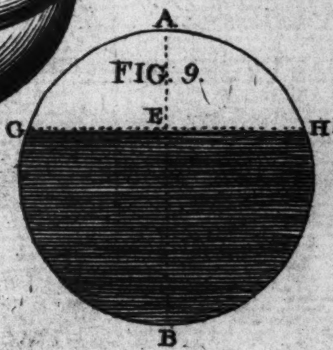
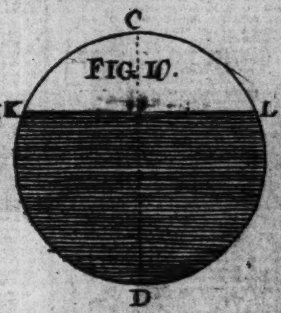


FIG. 10.



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‘ THE following rules are ſub‑ ‘ joined for the ready computation ‘ of the contents of veſſels, and of ‘ any ſolids in the meaſures in uſe ‘ in *Great Britain.*

‘ I. To find the content of a cy‑ ‘ lindric veſſel in *Engliſh* wine gal‑ ‘ lons, the diameter of the baſe and ‘ altitude of the veſſel being given ‘ in inches and decimals of an inch.

‘ Square the number of inches in ‘ the diameter of the veſſel; mul‑ ‘ tiply this ſquare by the number ‘ of inches in the height; then mul‑ ‘ tiply the product by the decimal ‘ fraction .0034; and this laſt pro‑ ‘ duct ſhall give the content in ‘ wine gallons and decimals of ſuch ‘ a gallon. To expreſs the rule a‑ ‘ rithmetically, let D repreſent the ‘ number of inches and decimals of ‘ an inch in the diameter of the ‘ veſſel, and H the inches and deci‑ ‘ mals of an inch in the height of ‘ the veſſel; then the content in wine gallons ſhall be $DDH \times \frac{34}{10000}$, or $DDH \times .0034$. *Ex.* Let the diameter $D = 51.2$ inches, the height $H = 62.3$ inches, then the content ſhall be $51.2 \times 51.2 \times 62.3 \times .0034 = 555.27342$ wine gallons. This rule follows from prop. 7. of the ſecond part, and prop. 3. of the third part; for by the former, the area of the baſe of the veſſel is in ſquare inches $DD \times .7854$; and by the latter, the content of the veſſel in ſolid inches is $DDH \times .7854$; which divided by 231 (the number of cubical inches in a wine gallon) gives $DDH \times .0034$ the content in wine gallons. But tho' the charges in the exciſe are made (by ſtatute) on the ſuppoſition that the wine-gallon contains 231 cubical inches, yet it is ſaid that in ſale 224 cubical inches (the content of the ſtandard meaſured in *Guildhall*, as was mentioned above)

above) are allowed to be a wine gallon.

‘ II. Suppoſing the *Engliſh* ale gallon to contain 282 cubical inches, the content of a cylindric veſſel is computed in ſuch gallons, by multiplying the ſquare of the diameter of the veſſel by its height as formerly, and their product by the decimal fraction .0027851. That is, the ſolid content in ale gallons is DDH × .0027851.

‘ III. Suppoſing the *Scots* pint to contain about 103.4 cubical inches (which is the meaſure given by the ſtandards at *Edinburgh*, according to experiments mentioned above) the content of a cylindric veſſel is computed in *Scots* pints, by multiplying the ſquare of the diameter of the veſſel by its height, and the product of theſe by the decimal fraction .0076. Or the content of ſuch a veſſel in *Scots* pints is DDH × .0076.

‘IV. Suppoſing the *Wincheſter* buſhel to contain 2178 cubical inches, the content of a cylindric veſſel is computed in thoſe buſhels by multiplying the ſquare of the diameter of the veſſel by the height, and the product by the decimal fraction .0003606. But the ſtandard buſhel having been meaſured by Mr. *Everard* and others in 1696, it was found to contain only 2145.6 ſolid inches; and therefore it was enacted in the Act for laying a duty upon malt, *That every round buſhel, with a plain and even bottom, being* 18½ *inches diameter throughout, and* 8 *inches deep, ſhould be eſteemed a legal* Wincheſter *buſhel.* According to this Act (ratified in the firſt Year of Queen *Anne*) the legal *Wincheſter* buſhel contains only 2150.42 ſolid inches. And the content of a cylindric veſſel is computed in ſuch buſhels, by multiplying the ſquare

‘ of the diameter by the height, and
‘ their product by the decimal fraction
‘ .0003652. Or the content
‘ of the veſſel in thoſe buſhels is
‘ DDH × .0003652.

‘ V. Suppoſing the *Scots* wheat
‘ firlot to contain $21\frac{1}{4}$ *Scots* pints,
‘ (as is appointed by the Statute
‘ 1618) and the pint to be conform
‘ to the *Edinburgh* ſtandards above
‘ mentioned, the content of a cylindric
‘ veſſel in ſuch firlots is computed
‘ by multiplying the ſquare
‘ of the diameter by the height, and
‘ their product by the decimal fraction
‘ .00358. This firlot, in 1426
‘ is appointed to contain 17 pints,
‘ in 1457 it was appointed to contain
‘ 18 pints, in 1587 it is $19\frac{1}{4}$
‘ pints, in 1618 it is $21\frac{1}{4}$ pints;
‘ and tho' this laſt Statute appears to
‘ have been founded on wrong computations
‘ in ſeveral reſpects, yet
‘ this part of the Act that relates to
‘ the number of pints in the firlot

‘ ſeems to be the leaſt exceptionable; and therefore we ſuppoſe the firlot to contain 21 ¼ pints of the *Edinburgh* ſtandard, or about 2197 cubical inches; which a little exceeds the *Winchester* buſhel, from which it may have been originally copied.

‘ Suppoſing the bear firlot to contain 31 *Scots* pints (according to the Statute 1618) and the pint conform to the *Edinburgh* ſtandards, the content of a cylindric veſſel in ſuch firlots is found by multiplying the ſquare of the diameter by the height, and this product by .000245.

‘ When the ſection of the veſſel is not a circle but an ellipſis, the product of the greateſt diameter by the leaſt is to be ſubſtituted in thoſe rules for the ſquare of the diameter.

‘ VII. To compute the content of a veſſel that may be conſidered

‘ as a *fruſtum* of a cone in any of thoſe meaſures.

‘ Let A repreſent the number of inches in the diameter of the greater baſe, B the number of inches in the diameter of the leſſer baſe. Compute the ſquare of A, the product of A multiplied by B, and the ſquare of B, and collect theſe into a ſum. Then find the third part of this ſum, and ſubſtitute it in the preceeding rules in the place of the ſquare of the diameter; and proceed in all other reſpects as before. Thus, for example, the content in wine gallons is $\overline{AA + AB + BB} \times \frac{1}{3} \times H \times .0036$.

‘ Or to the ſquare of half the ſum of the diameters A and B, add one third part of the ſquare of half their difference; and ſubſtitute this ſum in the preceeding rules for the ſquare of the diameter of the veſſel; for the ſquare of $\frac{1}{2}A + \frac{1}{2}B$ added to $\frac{1}{3}$ of the ſquare of $\frac{1}{2}A -$

‘ $\frac{1}{2}$ B, gives $\frac{1}{3}$ AA + $\frac{1}{3}$ AB + $\frac{1}{3}$ BB.

‘ VIII. When a veſſel is a *fruſtum* of a parabolic conoid, meaſure the diameter of the ſection at the middle of the height of the *fruſtum*; and the content will be preciſely the ſame as of a cylinder of this diameter, of the ſame height with the veſſel.

‘ IX. When a veſſel is a *fruſtum* of a ſphere, if you meaſure the diameter of the ſection at the middle of the height of the *fruſtum*, then compute the content of a cylinder of this diameter of the ſame height with the veſſel, and from this ſubſtract $\frac{1}{3}$ of the content of a cylinder of the ſame height on a baſe whoſe diameter is equal to its height; the remainder will give the content of the veſſel. That is, if D repreſent the diameter of the middle ſection, and H the height of the *fruſtum*, you are to ſubſtitute DD — $\frac{1}{3}$ HH for

‘ the ſquare of the diameter of the cylindric veſſel in the firſt ſix rules.

‘ X. When the veſſel is a *fruſtum* of a ſpheroid, if the baſes are equal, the content is readily found by the rule in *p.* 132. In other caſes, let the axis of the ſolid be to the conjugate axis, as *n* to 1; let D be the diameter of the middle ſection of the *fruſtum*, H the height or length of the *fruſtum*; and ſubſtitute in the firſt ſix rules $DD - \frac{HH}{3nn}$ for the ſquare of the diameter of the veſſel.

‘ XI. When the veſſel is an hyperbolic conoid, let the axis of the ſolid be to the conjugate axis, as *n* to 1, D the diameter of the ſection at the middle of the *fruſtum*, H the height or length, compute $DD + \frac{1}{3nn} \times HH$, and ſubſtitute this ſum for the ſquare of the diameter of the cylindric veſſel in the firſt ſix rules.

‘ XII. In general, it is uſual to

‘ meaſure any round veſſel, by diſtinguiſhing it into ſeveral *fruſtums*, and taking the diameter of the ſection at the middle of each *fruſtum*; thence to compute the content of each, as if it was a cylinder of that mean diameter; and to give their ſum as the content of the veſſel. From the total content computed in this manner they ſubſtract ſucceſſively the numbers which expreſs the circular areas that correſpond to thoſe mean diameters, each as often as there are inches in the altitude of the *fruſtum* to which it belongs, beginning with the uppermoſt; and in this manner calculate a table for the veſſel, by which it readily appears how much liquor is at any time contained in it, by taking either the dry or wet inches; having regard to the inclination or drip of the veſſel when it has any.

‘ This method of computing the

content of a *fruſtum* from the diameter of the ſection at the middle of its height is exact in that caſe only when it is a portion of a parabolic conoid; but in ſuch veſſels as are in common uſe, the error is not conſiderable. When the veſſel is a portion of a cone or hyperbolic conoid, the content by this method is found leſs than the truth; but when it is a portion of a ſphere or ſpheroid, the content computed in this manner exceeds the truth. The difference or error is always the ſame in the different parts of the ſame or of ſimilar veſſels when the altitude of the *fruſtum* is given. And when the altitudes are different, the error is in the triplicate *ratio* of the altitude. If exactneſs be required, the error in meaſuring the *fruſtum* of a conical veſſel, in this manner, is $\frac{1}{4}$ of the content of a cone, ſimilar to the veſſel, of an altitude equal to the

‘ height of the *fruſtum*. In a ſphere, it is $\frac{1}{3}$ of a cylinder of a diameter and height equal to the *fruſtum*. In the ſpheroid and hyperbolic conoid, it is the ſame as in a cone generated by the right-angled triangle contained by the two ſemi-axes of the figure revolving about that ſide which is the ſemi-axis of the *fruſtum*. Theſe are demonſtrated in a treatiſe of fluxions by Mr. *Mac Laurin*, *p.* 25. and 715. where thoſe theorems are extended to *fruſtums* that are bounded by planes oblique to the axis in all the ſolids that are generated by any conic ſection revolving about either axis.

‘ In the uſual method of computing a table for a veſſel, by ſubducting from the whole content the number that expreſſes the uppermoſt area as often as there are inches in the uppermoſt *fruſtum*, and afterwards the numbers for the

‘ other areas ſucceſſively, it is obvious that the contents aſſigned by the table, when a few of the uppermoſt inches are dry, are ſtated a little too high, if the veſſel ſtands on its leſſer baſe, but too low when it ſtands on its greater baſe; becauſe, when one inch is dry, for example, it is not the area at the middle of the uppermoſt *fruſtum*, but rather the area at the middle of the uppermoſt inch, that ought to be ſubducted from the total content, in order to find the content in this caſe.

‘ XIII. To meaſure round timber, let the mean circumference be found in feet and decimals of a foot; ſquare it, multiply this ſquare by the decimal .079577, and the product by the length. *Ex*. Let the mean circumference of a tree be 10.3 feet, and the length 24 feet. Then 10.3 × 10.3 × .079577 × 24 = 202.615 is the

‘ number of cubical feet in the tree.
‘ The foundation of this rule is, that
‘ when the circumference of a crcle
‘ is 1, the area is .079577471 5, and
‘ that the areas of circles are as the
‘ ſquares of their circumferences.

‘ But the common way uſed by
‘ Artificers for meaſuring round timber,
‘ differs much from this rule.
‘ They call one fourth part of the
‘ circumference the *girt*, which is
‘ by them reckoned the ſide of a
‘ ſquare, whoſe area is equal to the
‘ area of the ſection of the tree;
‘ therefore they ſquare the *girt*, and
‘ then multiply by the length of the
‘ tree. According to their method
‘ the tree of the laſt example would
‘ be computed at 159.13 cubical
‘ feet only. How ſquare timber is
‘ meaſured, will be eaſily underſtood
‘ from the preceeding propoſitions.
‘ Fifty ſolid feet of hewn
‘ timber, and forty of rough timber
‘ make a load.

‘ XIV. To find the burden of a ſhip, or the number of tons it will carry, the following rule is commonly given. Multiply the length of the keel taken within board, by the breadth of the ſhip within board, taken from the mid-ſhip beam from plank to plank, and the product by the depth of the hold, taken from the plank below the keelſon to the under part of the upper deck plank, and divide the product by 94, the quotient is the content of the tonnage required. This rule however cannot be accurate, nor can one rule be ſuppoſed to ſerve for the meaſuring exactly the burden of ſhips of all ſorts. Of this the reader will find more in the memoires of the Royal Academy of Sciences at *Paris* for the year 1721. ’

‘ Our Author having ſaid nothing of weights, it may be of uſe to

‘ add briefly that the *Engliſh* Troy-pound contains 12 ounces, the ounce twenty penny weight, and the penny weight 24 grains; that the *Averdupois* pound contains 16 ounces, the ounce 16 drams, and that 112 pounds is uſually called the hundred weight. It is commonly ſuppoſed that 14 pounds *Averdupois* are equal to 17 pounds Troy. According to Mr. *Everard's* experiments, one pound *Averdupois* is equal to 14 ounces, 11 penny weight and 16 grains Troy, that is, to 7000 grains; and an *Averdupois* ounce is 437 ½ grains. The *Scots* Troy pound, (which by the Statute 1718 was to be the ſame with the *French*) is commonly ſuppoſed equal to 15 ¾ ounces *Engliſh* Troy, or 7560 grains. By a mean of the ſtandards kept by the Dean of Gild of *Edinburgh*, it is 7599 $\frac{3}{32}$, or 7600 grains. They who have meaſur-

‘ ed the weights which were ſent
‘ from *London*, after the Union of
‘ the Kingdoms, to be the ſtandards
‘ by which the weights in *Scotland*
‘ ſhould be made, have found the
‘ *Engliſh Averdupois* pound (from a
‘ medium of the ſeveral weights)
‘ to weigh 7000 grains, the ſame as
‘ Mr. *Everard*; according to which
‘ the *Scots*, *Paris* or *Amſterdam*
‘ pound will be to the pound *Averdupois*
‘ as 38 to 35. The *Scots*
‘ Troy ſtone contains 16 pounds,
‘ the pound two marks or 16 ounces,
‘ an ounce 16 drops, a drop 36
‘ grains. Twenty *Scots* ounces make
‘ a Trone pound; but becauſe it is
‘ uſual to allow one to the ſcore, the
‘ Trone pound is commonly 21
‘ ounces. Sir *John Skene* however
‘ makes the Trone ſtone to contain
‘ only 19½ pounds.

FINIS.

MVSEVM
BRITAN
NICVM